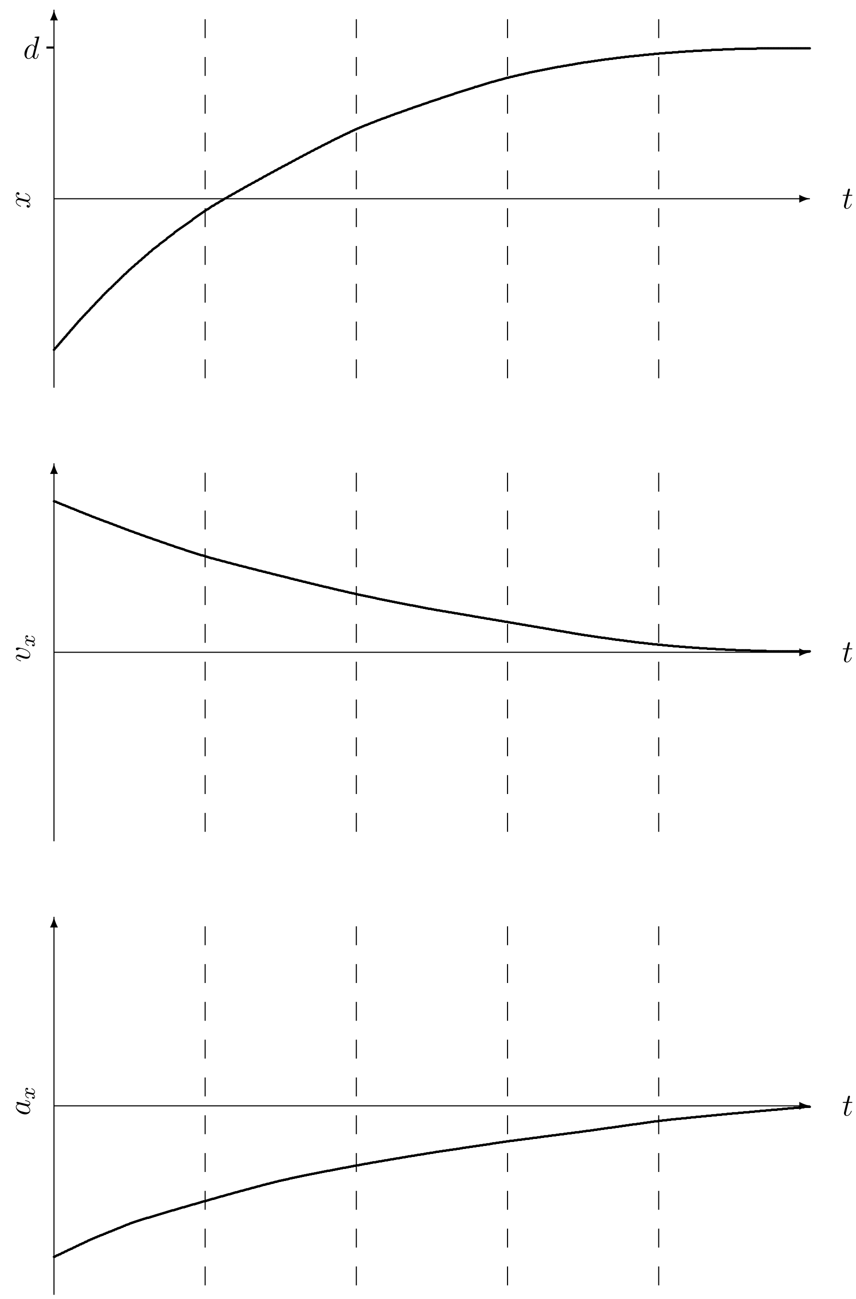
Solutions to Exam 1; Phys 185

1. (20 points) The top graph displays how position depends on time for an object that gradually, but ever more slowly, approaches $x = d$. Make a qualitative sketch of the corresponding velocity versus time and acceleration versus time graphs for this motion.



2. (30 points) You have a tall building, a spherical ball made of an aerogel material, and some lab equipment that allows you to measure the speed v of the ball just before it hits the ground. You can look up the density of air ρ, and the acceleration due to gravity g. You can measure the radius of the ball r, and the mass of the ball m.

(a) Describe (in one or two sentences) an experiment to measure the drag coefficient C_D on the ball. Find the equation you would use to calculate C_D.

Answer: Drop the ball from the building, measure the speed as it hits the ground. This will be it terminal speed, when the drag force is equal to the weight. Using

$$\tfrac{1}{2}C_D\rho A v^2 = mg \qquad \Rightarrow \qquad C_D = \frac{2mg}{\rho\pi r^2 v^2}$$

(b) What significance does the fact that the ball is made of an aerogel have? What significance does the fact that it is a sphere have?

Answer: The aerogel means that the ball is a very light object, so that it reaches its terminal speed very quickly. Dropping it off a tall building will give it plenty of time to approach very close to v_T. The spherical shape ensures that no matter how it falls, the cross-sectional area A is always the same, πr^2.

3. (40 points) You have a cannon that launches rubber balls with an initial speed of $v_i = 12.6$ m/s. You set it at an angle $\theta = 38°$ above the horizontal, and shoot a ball at a high vertical wall standing a distance $l = 9.20$ m in front of the cannon.

(a) Find v_{fx} and v_{fy} at the instant before the rubber ball hits the wall. (**Bonus +5 points** if you solve everything symbolically, only plugging in numbers at the very end.)

Answer: The initial velocity components are $v_{ix} = v_i \cos\theta$ and $v_{iy} = v_i \sin\theta$. The acceleration components are $a_x = 0$ and $a_y = -g$. We want to find v_{fx} and v_{fy} at the Δt when $x_f = l$. Therefore,

$$l = 0 + v_i \cos\theta + 0 \qquad \Rightarrow \qquad \Delta t = \frac{l}{v_i \cos\theta}$$

At this time,

$$v_{fx} = v_{ix} = v_i \cos\theta = 9.93 \text{ m/s}$$

and

$$v_{fy} = v_i \sin\theta - g\Delta t = v_i \sin\theta - \frac{gl}{v_i \cos\theta} = -1.32 \text{ m/s}$$

(b) The instant *after* the rubber ball bounces off the wall, the y-component of its velocity remains the same as it was just before it hit the wall. But the x-component of its velocity reverses its direction (same magnitude, opposite sign). Find out where, relative to the cannon, the ball falls back to the ground.

Answer: There are multiple ways to solve this. The easiest is to recognize that reversing v_x means that the motion after the ball hits the wall will be the same as if the wall were not there, but in the $-x$ direction instead. The range without the wall would be

$$R = \frac{2v_i^2 \sin\theta\cos\theta}{g}$$

so the ball would have traveled an extra distance of $R-d$. Since the direction is reversed, we *subtract* that from d, find in that ball landed a distance

$$d-(R-d) = 2d - R = 2d - \frac{2v_i^2 \sin\theta\cos\theta}{g} = 2.7 \text{ m}$$

in front of the cannon.

4. (30 points) You set up a device where you have a bucket tethered to a rope, and the bucket rotates at a constant speed v in a vertical circle with radius r. You then place a small ball with mass m in the bucket. What is the minimum v you must have in order for the ball not to fall out of the bucket during rotation?

Answer: At the top of the circle, if the speed is just barely enough to keep the ball from falling, the normal force on the ball will be zero. The acceleration will need to be directed downwards (toward the center of the circle), and have a magnitude of v^2/r. Therefore

$$\sum F_y = ma_y \qquad \Rightarrow \qquad w + n = \frac{mv^2}{r}$$

With $n = 0$ for $v = v_{\mathsf{min}}$,

$$mg = \frac{mv_{\mathsf{min}}^2}{r} \qquad \Rightarrow \qquad v_{\mathsf{min}} = \sqrt{gr}$$

5. (80 points) In Lab 3, "Acceleration due to Gravity," you set two light gates a distance Δx apart, and measured the time the cart took, Δt, to cover that distance.

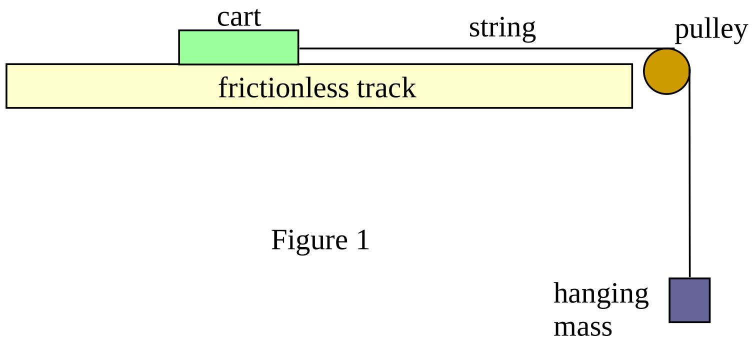


Figure 1

A source of error was difficulty setting the initial velocity at the first light gate. So you persuade the physics department to spend money for a mechanism so that $v_i = 0.000$ m/s.

(a) In the lab, you saw that the acceleration of the cart was $a = 2\Delta x/(\Delta t)^2$, under the assumption that $v_i = 0$ at the first light gate. Derive this equation.

Answer: Since $v_i = 0$ and $\Delta x = x_f - x_i$, we have

$$x_f = x_i + 0 + \tfrac{1}{2}a(\Delta t)^2 \qquad \Rightarrow \qquad a = \frac{2\Delta x}{(\Delta t)^2}$$

(b) Repeat the calculation we did in class, and use it to derive the equation for g we used in the lab—find the equation for g in terms of Δx, Δt, m_{hang}, and m_{cart}.

Answer: The hanging mass and the cart will have the same magnitude of acceleration, a, but in different directions due to the pulley.

On the hanging mass,

$$\sum F_x = 0 + 0 = m_{\text{hang}}0 = 0$$

$$\sum F_y = T - m_{\text{hang}}g = -m_{\text{hang}}a$$

On the cart,

$$\sum F_x = T = m_{\text{cart}}a$$

$$\sum F_y = n - m_{\text{cart}}g = -m_{\text{cart}}0 = 0$$

We substitute $T = m_{\text{cart}}a$ in the other equation with T and get

$$m_{\text{cart}}a - m_{\text{hang}}g = -m_{\text{hang}}a$$

which means

$$g = \frac{(m_{\text{cart}} + m_{\text{hang}})}{m_{\text{hang}}}a = \frac{(m_{\text{cart}} + m_{\text{hang}})}{m_{\text{hang}}}\frac{2\Delta x}{(\Delta t)^2}$$

(c) You now realize that friction is another source of error, and that the physics department is now out of money to buy tracks with nearly no friction. Say you know that the coefficient of rolling friction between the cart and the track is μ_r. What would your altered equation for g then be, accounting for friction?

Answer: Friction only adds another force to the cart. Noticing that on the cart, the $\sum F_y = 0$ equation results in $n = m_{\text{cart}}g$, and that $f_r = \mu_r n = \mu_r m_{\text{cart}}g$, the only equation that changes is

$$\sum F_x = T - \mu_r m_{\text{cart}}g = m_{\text{cart}}a$$

This now gives

$$m_{\mathsf{cart}}a - m_{\mathsf{hang}}g + \mu_r m_{\mathsf{cart}}g = -m_{\mathsf{hang}}a$$

which means

$$g = \frac{(m_{\mathsf{cart}} + m_{\mathsf{hang}})}{(m_{\mathsf{hang}} - \mu_r m_{\mathsf{cart}})}a = \frac{(m_{\mathsf{cart}} + m_{\mathsf{hang}})}{(m_{\mathsf{hang}} - \mu_r m_{\mathsf{cart}})}\frac{2\Delta x}{(\Delta t)^2}$$

(d) But your track might not be perfectly level! Say it has a tilt of θ with respect to the horizontal. What would your altered equation for g then be, accounting for everything?

Answer: Now use tilted x and y-axes for the table. The only difference will be that w will now have an x and y component: $w_x = m_{\mathsf{cart}}g\sin\theta$ and $w_y = -m_{\mathsf{cart}}g\cos\theta$. On the cart,

$$\sum F_x = T + m_{\mathsf{cart}}g\sin\theta - f_r = m_{\mathsf{cart}}a$$

$$\sum F_y = n - m_{\mathsf{cart}}g\cos\theta = -m_{\mathsf{cart}}0 = 0$$

Accounting for the small changes, such as $n = m_{\mathsf{cart}}g\cos\theta$ and $f_r = \mu_r m_{\mathsf{cart}}g\cos\theta$, the algebra becomes

$$m_{\mathsf{cart}}a - m_{\mathsf{hang}}g + \mu_r m_{\mathsf{cart}}g\cos\theta - m_{\mathsf{cart}}g\sin\theta = -m_{\mathsf{hang}}a$$

which means

$$g = \frac{(m_{\mathsf{cart}} + m_{\mathsf{hang}})}{(m_{\mathsf{hang}} - \mu_r m_{\mathsf{cart}}\cos\theta + m_{\mathsf{cart}}\sin\theta)}a = \frac{(m_{\mathsf{cart}} + m_{\mathsf{hang}})}{(m_{\mathsf{hang}} - \mu_r m_{\mathsf{cart}}\cos\theta + m_{\mathsf{cart}}\sin\theta)}\frac{2\Delta x}{(\Delta t)^2}$$