Solutions to Exam 1; Phys 185

1. (30 points) A person with compromised pinch strength in their fingers can only exert a normal force of magnitude n to either side of a pinch-held object. Find an equation for $m_{\max}$, the mass of the heaviest book this person can hold onto vertically before it slips out of his or her fingers, in terms of n, g, and μ_s.

Answer: There is a normal force on each side along the x-axis, one static friction force on each side, and the weight of the book. So we have

$$n_{1x} = n_1 \qquad n_{2x} = -n_2$$

$$f_{s1y} = f_{s1} \qquad f_{s2y} = f_{s2} \qquad w_y = -w$$

and all the other force components are zero. The book is stationary, so $a_x = a_y = 0$. This means

$$\sum f_x = n_1 - n_2 = 0 \qquad \Rightarrow \qquad n_1 = n_2 = n$$

$$\sum f_y = f_{s1} + f_{s2} - w = 0 \qquad \Rightarrow \qquad f_{s1} + f_{s2} = w$$

Now, use $w = mg$. Since we're looking for the heaviest book, $f_{s1} = f_{s1,\max} = \mu_s n$. We found that $n_1 = n_2 = n$, therefore $f_{s2} = \mu_s n_1$ also. We end up with

$$2\mu_s n = m_{\max} g \qquad \Rightarrow \qquad m_{\max} = \frac{2\mu_s n}{g}$$

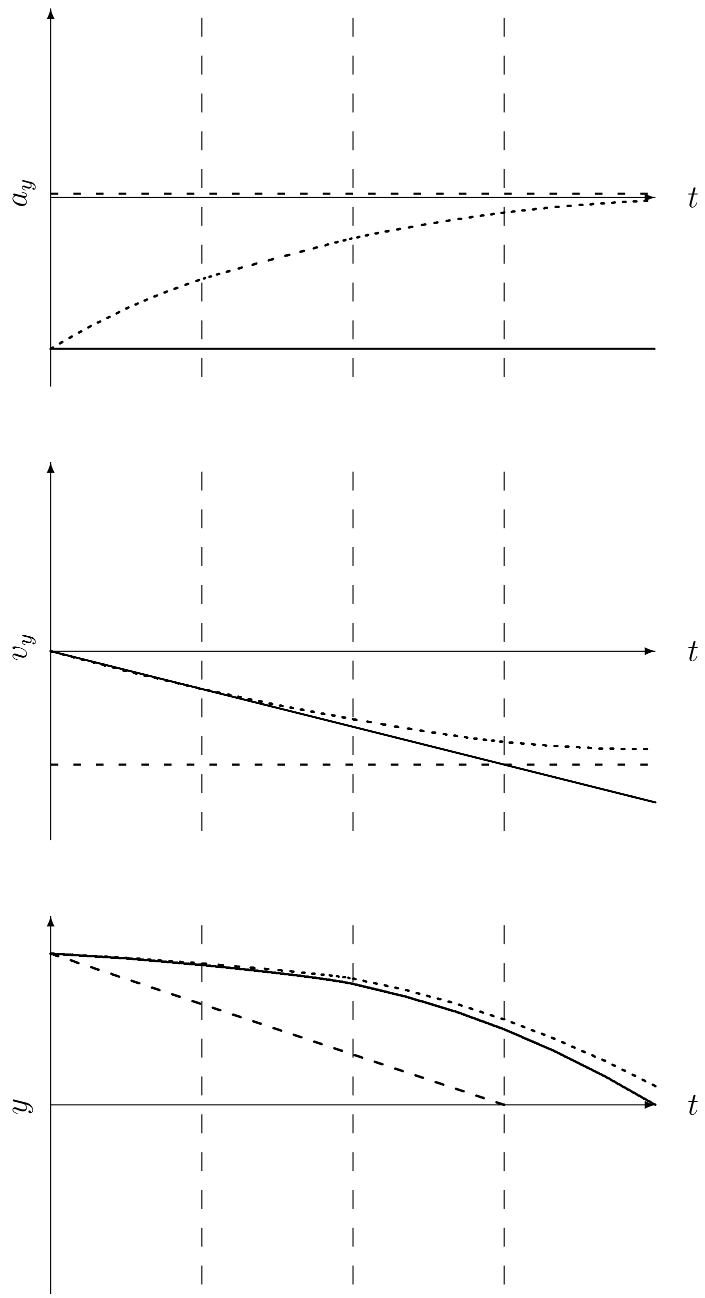
2. (50 points) Make qualitative sketches of the acceleration, velocity and position for the three following descriptions of a falling object. All three have the same initial position y_i.

(a) Solid line: The object has only gravity acting on it. It starts at rest: $v_{iy} = 0$.

(b) Dotted line: The object has gravity plus a significant drag force acting on it. $v_{iy} = 0$.

(c) Dashed line: The object moves straight down with a *constant* speed equal to v_T, the terminal speed in the presence of gravity and air resistance. $v_{iy} = v_T$.

The exact shapes of the curves are not important, but be clear about straight lines and points of intersection. *In the space next to each graph, explain your reasoning.*



Answer:

3. (40 points) You launch a projectile on a level surface on a planet with acceleration due to gravity g, starting from $x_i = y_i = 0$, with initial speed v_i and angle θ with the x-axis. But you're facing a strong horizontal wind, so that the motion has a non-zero $a_x = -w$, where w is a positive constant that stands for the magnitude of the acceleration due to the wind.

(a) Write down the equations for motion along the x and y-axes:

$$v_{fx} = v_i \cos\theta - w\Delta t \qquad x_f = v_i \cos\theta \Delta t - \tfrac{1}{2}w(\Delta t)^2$$

$$v_{fy} = v_i \sin\theta - g\Delta t \qquad y_f = v_i \sin\theta\Delta t - \tfrac{1}{2}g(\Delta t)^2$$

(b) Find the *range* of the projectile: an equation for how far it will travel until it hits the ground again.

Answer: You want Δt for $y_f = 0$. The wind has no effect on this; you end up with the usual

$$0 = v_i \sin\theta\Delta t - \tfrac{1}{2}g(\Delta t)^2 \qquad \Rightarrow \qquad \Delta t = 0 \text{ or } \frac{2v_i \sin\theta}{g}$$

where you throw away the $\Delta t = 0$ solution. The range is x_f at this Δt:

$$x_f = v_i \cos\theta \frac{2v_i \sin\theta}{g} - \tfrac{1}{2}w\left(\frac{2v_i \sin\theta}{g}\right)^2 = \frac{2v_i^2 \sin\theta}{g}\left(\cos\theta - \frac{w}{g}\sin\theta\right)$$

(c) Check your result: when you set $w = 0$, you should get the same equation for the range as you have in your class notes.

Answer: With $w = 0$, you get

$$x_f = \frac{2v_i^2 \sin\theta\cos\theta}{g}$$

which is what you should have.

(d) The range is positive when $w <$ [an expression involving g and θ]. Find this inequality. Would it make physical sense for the range to be negative?

Answer: Looking at the equation, you see that $x_f > 0$ when

$$\left(\cos\theta - \frac{w}{g}\sin\theta\right) > 0 \qquad \Rightarrow \qquad w < g\cot\theta$$

The wind can be strong enough that the projectile loops backward.

(e) See what happens when $w = g$ and $\theta = 45°$. Interpret your result in this case—what does the motion look like?

Answer: In this case the range ends up as zero. The total acceleration vector is toward the origin at at a 45° angle—you shoot your projectile straight into that; it goes diagonally up a bit and comes straight back down.

4. (40 points) An adult falls from a height h, and then strikes a pad with thickness d. Model the the person's contact with the pad as motion with constant acceleration that brings her to a complete stop after traveling a distance d. Say that damage to the body becomes probable when its stopping acceleration exceeds $10g$.

(a) What is the minimum ratio d/h for safety?

Answer: Falling from h, the person hits the pad at time

$$0 = h - \tfrac{1}{2}g(\Delta t)^2 \qquad \Rightarrow \qquad \Delta t = \sqrt{2h/g}$$

At this time, the velocity will be

$$v_{fy} = -g\Delta t = -\sqrt{2hg}$$

This is not the initial velocity into the pad. To travel d to come to a stop, we have

$$0 = -\sqrt{2hg} + a\Delta t \qquad \Rightarrow \qquad \Delta t = \frac{\sqrt{2hg}}{a}$$

Putting this in the equation for the distance traveled,

$$0 = d - \sqrt{2hg}\frac{\sqrt{2hg}}{a} + \tfrac{1}{2}a\frac{2hg}{a^2} \qquad \Rightarrow \qquad d = \frac{hg}{a}$$

Therefore

$$a = \frac{h}{d}g$$

Since $a = 10g$ is the limit, this means

$$\frac{d}{h} = \frac{1}{10}$$

(b) For a three story height, $h \approx 10$ m. About what thickness of landing pad would be needed? Is this a reasonable number?

Answer: 1 m. For a rough calculation, this seems to be reasonable.

5. (40 points) In Lab 3, "Acceleration due to Gravity," you made a number of simplifying assumptions. In the list below, describe how neglecting each effect made you slightly underestimate or overestimate g as calculated from your data. (I include the answer for the first as an example.) Also draw and label arrows for *every* force you know of on the diagram, including those that you neglected as being too small to worry about, and including forces on the string and pulley.

(a) **Track not frictionless:** A small extra kinetic friction force on the cart toward the left slows the cart down, increasing the measured Δt. The equation for g was

$$g = \frac{2(m_{\mathsf{hanging}} + m_{\mathsf{cart}})\Delta x}{m_{\mathsf{hanging}}\,(\Delta t)^2}$$

With increased Δt, friction will cause us to underestimate g.

(b) **Drag on the cart:** Though not constant, drag, like friction, opposes the motion. Its effect will be same, leading to an underestimate of g.

(c) **Track not exactly level; cart goes slightly downhill:** A downhill tilt will add a component of the weight in the forward direction, making the cart go faster and reducing the measured Δt. So the effect is the opposite of friction: it will lead to an overestimate of g.

(d) **Drag on hanging mass:** This drag will be in the same direction as the tension in the string, therefore reducing the tension opposing the weight of the hanging mass. The same tension pulls the cart, which will now be smaller. This increases Δt and leads to an overestimate of g.

(e) **Initial speed of cart into first photogate not quite zero:** Being faster will reduce Δt and lead to an underestimate of g.

(f) **Pulley not frictionless:** Friction in the pulley will also increase Δt and lead to an overestimate of g.

(g) **Pulley not massless:** This means that a force is needed to accelerate the rotation of the pulley. This means the string tension on either side of the pulley cannot be equal: the cart side must be slightly less in magnitude than that on the hanging mass side. Reduced tension increases Δt and leads to an overestimate of g.