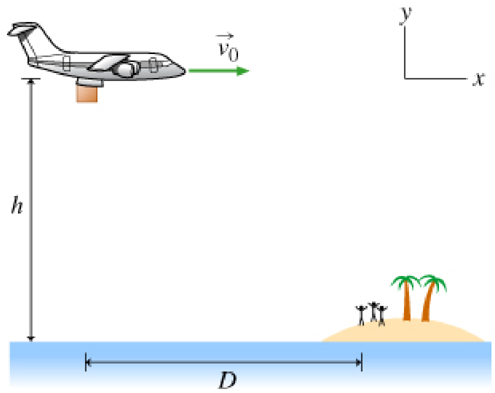
Solutions to Exam 1; Phys 185

1. (30 points) A relief airplane is dropping a food package to a group of people stranded on a very small island. The airplane flies horizontally with constant speed of v_0 at an altitude of h. The acceleration due to gravity is g. If the package is to land right on the island, at what horizontal distance D from the plane to the island should the package be released?



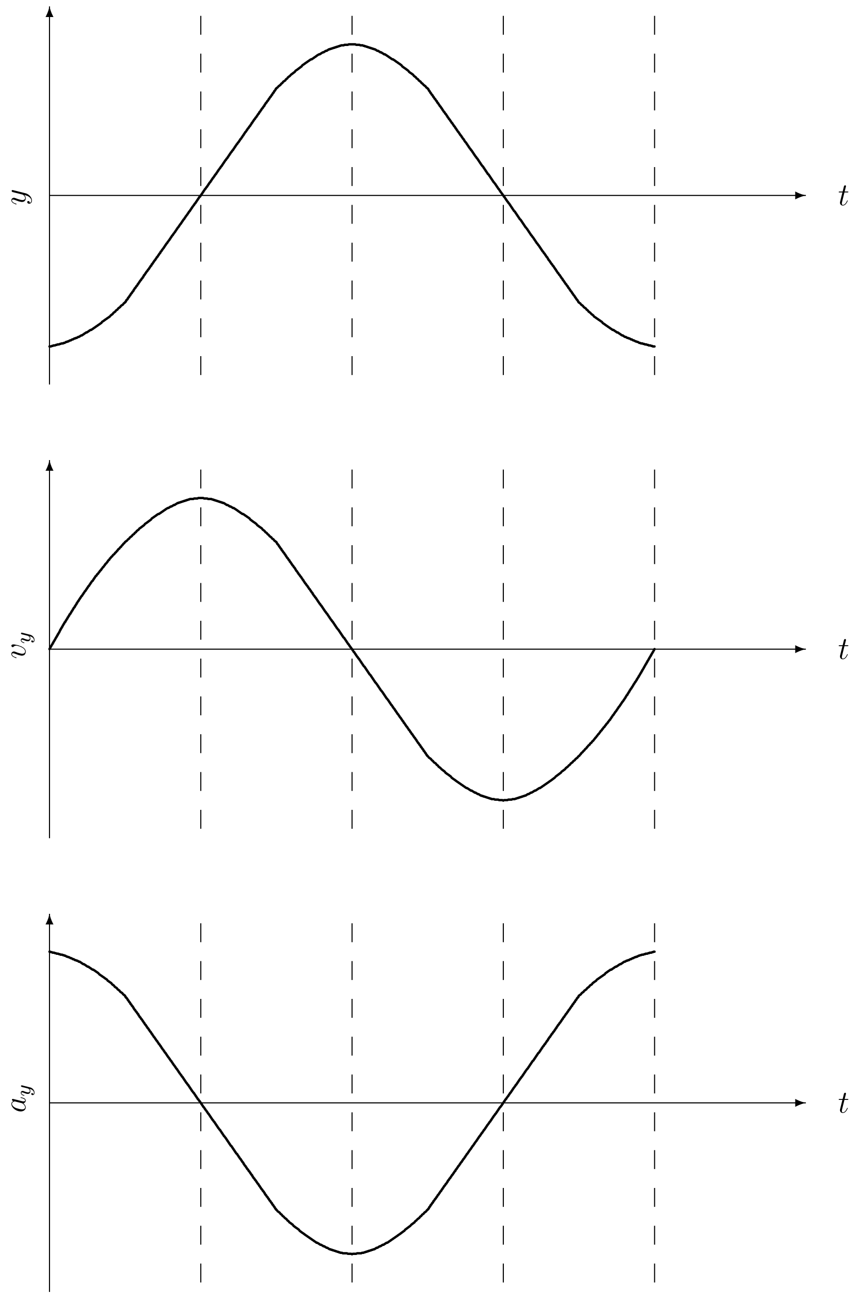
Answer: Horizontal flight means $v_{ix} = v_0$ and $v_{iy} = 0$. To reach $y_f = 0$ starting from $y_i = h$ takes

$$0 = h + v_{iy}\Delta t - \tfrac{1}{2}g\,(\Delta t)^2 \qquad \Rightarrow \qquad \Delta t = \sqrt{\frac{2h}{g}}$$

During that time, since the horizontal acceleration is zero, the package will travel a horizontal distance

$$D = v_{ix}\Delta t = v_0\sqrt{\frac{2h}{g}}$$

2. (40 points) The middle graph shows the velocity of an oscillating mass attached to a spring (as in Lab 5) over a full period. At $t = 0$, the spring is stretched to its maximum extent downward; you let go and the mass moves upward. $y = 0$ is the position for an unextended spring. Sketch the position and acceleration graphs as accurately as you can.



y
t
v_y
t
a_y
t

3. (60 points) You're doing the experiment in Lab 2, with the cart going up and down an inclined low-friction track. You notice that the accelerations going up and down are slightly different; $a_{\mathsf{up}} = -2.1\ \text{m/s}^2$ and $a_{\mathsf{down}} = -1.9\ \text{m/s}^2$; where these are accelerations along an x-axis tilted to be parallel with the track, with the $+x$-direction pointing up (away from the motion detector).

(a) Find θ, the angle at which the track is tilted. *Hint:* If you solve this symbolically, you'll find that θ depends on $\frac{1}{2}(a_{\mathsf{up}} + a_{\mathsf{down}})$ and g.

Answer: The only difference between the up and down motions is that the kinetic friction force reverses direction, since it points opposite the velocity. The weight and normal forces remain the same. Going up,

$$\sum F_y = w_y + n_y = -mg\cos\theta + n = 0 \qquad \Rightarrow \qquad n = mg\cos\theta$$

$$\sum F_x = w_x + f_{kx} = -mg\sin\theta - \mu_k mg\cos\theta = ma_{\mathsf{up}} \qquad \Rightarrow \qquad a_{\mathsf{up}} = -g(\sin\theta + \mu_k\cos\theta)$$

Coming down,

$$\sum F_y = w_y + n_y = -mg\cos\theta + n = 0 \qquad \Rightarrow \qquad n = mg\cos\theta$$

$$\sum F_x = w_x + f_{kx} = -mg\sin\theta + \mu_k mg\cos\theta = ma_{\mathsf{down}} \qquad \Rightarrow \qquad a_{\mathsf{down}} = -g(\sin\theta - \mu_k\cos\theta)$$

If we add the equations for a_{up} and a_{down}, we get

$$a_{\mathsf{up}} + a_{\mathsf{down}} = -2g\sin\theta \qquad \Rightarrow \qquad \theta = \sin^{-1}\left[-\left(\frac{a_{\mathsf{up}} + a_{\mathsf{down}}}{2}\right)\frac{1}{g}\right] = 12^\circ$$

(b) Find μ_k, the coefficient of kinetic friction between the track and the cart. *Hint:* If you solve this symbolically, you'll find that μ_k depends on $\frac{1}{2}(a_{\mathsf{up}} - a_{\mathsf{down}})$, g, and θ.

Answer: If we now subtract the equations for a_{up} and a_{down}, we get

$$a_{\mathsf{up}} - a_{\mathsf{down}} = -2\mu_k g\cos\theta \qquad \Rightarrow \qquad \mu_k = -\left(\frac{a_{\mathsf{up}} - a_{\mathsf{down}}}{2}\right)\frac{1}{g\cos\theta} = 0.010$$

This is quite small, as befits a low friction track.

4. (30 points) Say you drop a metal sphere with mass m under water, where the drag force is $\vec{D} = -c\vec{v}$, with c a constant. Aside from drag and weight, it also has an upward buoyant force $\vec{F}_B$ acting on it, which has magnitude $F_B = m_w g$, where m_w is the mass of water with a volume equal to that of the metal sphere, and $m_w < m$. Would the sphere attain a terminal speed, just as in air? If you think it will, derive an equation for v_T in terms of c, m, m_w, and g. Otherwise, if you think it won't, draw a rough graph of v vs t for an object falling in liquid, starting from rest.

Answer: Yes, the same reasoning for terminal speed in air applies. When the sphere is released, it's at rest, therefore the drag on it is zero. It accelerates downward. The weight and buoyant forces, however, are constant, and as the sphere speeds up, the drag on it increases, therefore decreasing the total downward force and making the acceleration approach zero. Terminal speed happens when all the forces cancel each other out:

$$\sum F_y = D - w + F_B = cv_T - mg + m_w g = 0 \qquad \Rightarrow \qquad v_T = \frac{(m - m_w)g}{c}$$

5. (40 points) You have an object with mass m moving on a flat surface, released with initial velocity v_i. The coefficient of kinetic friction between the surface and the mass is μ_k.

(a) Find the distance the object will travel before coming to a halt, in terms of v_i, μ_k, m, and g.

Answer: Adding up the forces will give the acceleration components:

$$\sum F_y = n - mg = ma_y = 0 \qquad \Rightarrow \qquad n = mg$$

$$\sum F_x = -f_k = -\mu_k n = -\mu_k mg = ma_x \qquad \Rightarrow \qquad a_x = -\mu_k g$$

Notice that the mass cancels out. With constant acceleration, we have $v_f = 0$ and

$$0 = v_i - \mu_k g \Delta t \qquad \Rightarrow \qquad \Delta t = \frac{v_i}{\mu_k g}$$

$$\Delta x = v_i \Delta t - \tfrac{1}{2}\mu_k g \left(\Delta t\right)^2 = \frac{v_i^2}{2\mu_k g}$$

(b) You also have a second object which is released with the same initial velocity v_i. This second object is placed in a sleeve that reduces friction by 1%, so that its coefficient of kinetic friction with the surface is $0.99\mu_k$, but increases its total mass by 1%, so that its mass is $1.01m$. Which object will travel a larger distance before coming to a halt?

Answer: The force calculation is exactly the same, including the increased mass canceling out. So we use the same results, only replacing μ_k with $0.99\mu_k$. Therefore

$$a_x = -0.99\mu_k g \qquad \text{and} \qquad \Delta x = \frac{v_i^2}{1.98\mu_k g}$$

This Δx is smaller than the result in (a); the second object travels a larger distance.