Questions before Exam 2

1. **As the distance d_{12} between two masses m_1 and m_2 increases, the magnitude of the attractive gravitational force between them becomes smaller, while their gravitational potential energy becomes larger. How is this possible? What happens at very large distances?**

   The force magnitude falls off as the square of distance:

   $$F_g = G\frac{m_1 m_2}{d_{12}^2}$$

   This gets smaller, but more and more gradually as d_{12} increases. As $d_{12} \to \infty$, $F_g \to 0$.

   The potential energy expression is:

   $$U_g = -G\frac{m_1 m_2}{d_{12}}$$

   Note the $-$ sign in front. This means that U_g is negative, and it *increases* as d_{12} gets larger. As $d_{12} \to \infty$, $U_g \to 0$.

   Question I could ask: For objects close to the surface of the Earth that are gravitationally attracted to the Earth, $d_{12} \approx R_E$ and $m_1 = m_E$, where R_E and m_E are the radius and mass of the Earth. Say you take a mass m close to the surface and raise it by a height h. What is the increase in the gravitational potential energy of the mass? Use the mathematical approximation that, since $R_E \gg h$,

   $$\frac{1}{R_E + h} \approx \frac{1}{R_E}\left(1 - \frac{h}{R_E}\right)$$

2. **What about black holes?**

   If the gravitational attraction by a mass is so strong that not even light can escape, that is a black hole.

More precisely, a spherical mass of radius r and mass m has an "escape speed" or "escape velocity" v_e associated with it: it is the minimum speed by which you can launch an object from its surface so that it will continue traveling away forever, until the distance d becomes ∞. This speed v_e is easily found by using energy conservation.

Question I could ask: Calculate the radius of a black hole of mass m. This is r for which $v_e = c$, where c is the speed of light. If you are closer to a black hole than this r, you can never escape, because not even light can.

3. **Why are planets closer to the sun smaller than those that are far?**

   The planet formation process involves a spinning disk of material. Gravitational attraction leads to collapse and the formation of a central star, but conservation of angular momentum means that the star could not form if *all* the material went into the star. Its moment of inertia would be very low, since the mass would be concentrated very close to the axis of rotation, and therefore to have the same $L = I\omega$, with a tiny I, ω would have to be huge—the star would rotate so fast it would fall apart.

   The planets in a solar system have a small mass, but large distance from the axis of rotation, so their $I = mr^2$ is appreciable. Indeed, most of the angular momentum of the solar system will be associated with the planets. Gas giants with large mass and a large distance to the star, such as Jupiter, carry most of the angular momentum. Small, rocky planets closer to a star are essentially byproducts of the formation process: they can't carry a lot of angular momentum.

   Question I could ask: Question 3 in the 2014 Exam 2.

4. **We have two identical spheres with identical hamsters inside, both rolling without slipping down an incline. In one of them, the hamster clings to the inside surface and rolls with the sphere; in the other, the hamster runs along the inside and doesn't roll. If released from the same height, which sphere reaches the bottom faster?**

   Question I could ask: This question. Present your reasoning in mathematical terms, as much as you can.

5. **How does a gyroscope work?**

   A mechanical gyroscope is a spinning object, typically a circle or disk, mounted in such a way that it has little contact with its surroundings. Due to angular momentum conservation, the gyroscope will maintain its axis of rotation—its orientation—so it's very useful to keep track of directions.

   Question I could ask: Why is it easier to keep a bicycle upright when it's moving?

6. **How can cats rotate themselves in the air while falling—doesn't that violate angular momentum conservation?**

   Here's a very recent news story about this very question.

   Question I could ask: Our carts on the tracks in the lab: they are mounted on wheels, and the carts slow down a little due to friction. Therefore their wheels also slow down. Why is the angular momentum of the turning wheels not conserved?

7. **How does angular momentum conservation work for rotating objects in space interacting in complicated ways?**

   One complication is that we need the 3D version of angular momentum, while an introductory course like this only deals with angular momentum with a fixed axis of rotation, and hence 2D. Setting that aside, the most interesting aspect of angular momentum conservation with multiple extended objects is that the *total* angular momentum includes both the angular momentum due to objects orbiting around one another, and the angular momentum due to the objects spinning around their own axes.

   For example, the Earth-Moon system has angular momentum due to the Earth and the Moon spinning, and the orbital angular momentum due to them revolving around their common center of mass. It's the total of *all* these angular momenta that is conserved. Due to tidal interactions between the Earth and Moon, the rate of rotation of the Earth very gradually slows down over time. So during the time of the dinosaurs, the Earth day was a bit shorter than the 24 hours we have today. Due to angular momentum conservation, this change in the spin angular momentum of the Earth has to be compensated by a change in the orbital angular momentum of the Moon. A change in ω for the Moon also means a change in the distance between the Earth and Moon, since a satellite cannot speed or slow down and maintain the same orbital radius.

Question I could ask: During the dinosaurs' time, did the Moon look larger or smaller in the sky compared to today?

8. **On a roller coaster, what kind of energy transformations occur? What are the conditions for not falling off a completely circular loop?**

   Roller coaster rides start high, with plenty of gravitational potential energy to transform into kinetic energy. Throughout the ride, energy is continually lost to friction and drag as well.

   To make it through a loop, the roller coaster carts have to be fast enough. At the very top, the weight of the cart plus the normal force from the track have to add together to provide the total force giving the centripetal mv^2/r. The normal force can only be downward. So if you find the conditions in which the normal force has to be upward to keep the roller coaster on the track, that's when it can't complete the loop.

   Question I could ask: Question 4 in the 2014 Exam 2.

9. **What are the energies involved in seismic events such as earthquakes?**

   The closest to what we have learned about so far is the energy associated with the elasticity of rock, which is very like the spring force at a first approximation. When compressed or otherwise stressed, rocks, like just about every object around us, act like springs; in this case, springs with a very high stiffness or k. With a large k, even small amounts of compression x can correspond to large amounts of energy $\frac{1}{2}kx^2$.

   Question I could ask: You have two identical springs with spring constant k. You attach them back-to-back and compress the whole arrangement by a total amount x. How much total energy will be stored in the two-spring arrangement?

10. **What conservation laws are involved in nuclear fission?**

Energy, linear momentum, and angular momentum conservation are still involved. However, energy and momentum have to be expressed in modified, relativistic forms; after all, fission involves some conversion of mass into energy ($E = mc^2$). And angular momentum conservation will also involve quantum mechanical intrinsic angular momentum, called "spin."

Then there is charge conservation, conservation of baryon and lepton numbers, etc. etc.

Question I could ask: Early in the 20th century, physicists discovered "beta decay," in which a neutron changed into a proton plus an electron: $n \to p + e^-$. But they found that such decays always had missing energy, and it looked like they didn't conserve momentum. So they thought there must be a very hard-to-detect particle called a neutrino involved, which carried the missing momentum and energy: $n \to p + e^- + \bar{\nu}_e$. But are conservation laws really legitimate if whenever you detect a violation you get to invent an invisible particle?