

Note: As I walk around, you can ask me for help; for example, to supply an equation or a number you have forgotten down, or to give you algebra aid. If you do, however, I will write down what help I provided on your exam, and grade your answer accordingly. It costs nothing to have me check and tell you if your answer on a question is correct or not.

1. (40 points) Why are brakes on a bicycle applied to the rim of a wheel and not on the axle? Model the bicycle wheel as a disk. Apply the same normal force with the brake pad, with the brake pad placed either near the rim or near the axle. The materials that come into contact are the same in both cases. In which case would the wheel come to rest faster? Construct a full argument, using equations as appropriate.

2. (40 points) You have two masses m_1 and m_2 , separated by a distance d , in outer space far from anything else. They both revolve, with the same angular velocity ω , about their common center of mass, which is stationary. Take the positions to be $x_1 = y_1 = y_2 = 0$ and $x_2 = d$.

- (a) Show that in this case, the total linear momentum is zero and remains zero. (In other words, if the masses rotate about any point other than the center of mass, momentum won't be conserved.) *Hint:* For uniform circular motion, $v = \omega r$.

- (b) Now assume that $m_1 \gg m_2$, so that $m_2/(m_1 + m_2) \approx 0$. (This is like a black hole and a small star.) Find an equation for m_1 in terms of d , ω , and the gravitational constant G .

3. (40 points) Use the astronomical data in the back flap of your textbook for the following:

(a) About where is the axis of rotation of our solar system located? Is this exact, or an approximation? Explain.

(b) Calculate the moments of inertia of the Sun, Earth, and Jupiter. Be explicit about what approximations you are using to get the moment of inertia for each.

(c) Calculate the angular momentum of the Sun, Earth, and Jupiter. (The Sun's period of rotation around its own axis is 24.5 days.) Add all these angular momenta together to get a total, and state what percentage of this total is associated with each.

4. (40 points) Find v_{min} , the minimum speed a roller coaster must have at the bottom of a circular loop, if it is to make it all the way around a loop with radius r . Assume the track is frictionless.

5. (40 points) The Earth is in orbit around the sun, and you can calculate its speed v_E at any moment. A comet with mass $m = 0.001m_E$ and speed v but going in the exact opposite direction approaches the Earth.

- (a) Say the comet collides head-on with the Earth. Assume no significant material gets ejected into space. What must v be to produce a 1% change in the speed of the Earth?

- (b) Say the comet misses the Earth entirely, but interacts with Earth's gravity so that it gets a "slingshot" so that it ends up going straight back in the direction it came from. If there is, again, a 1% reduction in the speed of the Earth, what must the initial and final speeds of the comet be?