Solutions to Exam 3; Phys 185

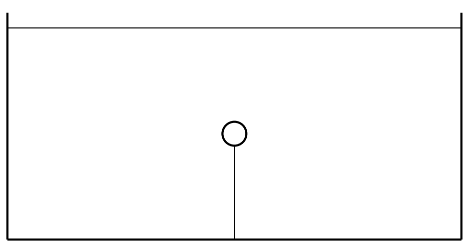
1. (20 points) Explain why heat flows from hot to cold but not the other way around.

Answer: In your collision experiment with carts in the lab, you saw energy being transferred from a moving (high kinetic energy) cart to the one initially at rest (low kinetic energy).

When you bring a hot and cold object together, they interact by particle collisions at the interface. Remember, temperature is a statistical quantity. So the particles on the hot side are more likely to have higher kinetic energy than the particles on the cold side. Therefore in most collisions, energy will be transferred to the cold side, heating it up while cooling the hot side down. This will continue until the temperatures are equalized, at which point on average no energy is transferred back and forth.

Since we are talking about statistical descriptions, there is a nonzero probability of heat flowing from cold to hot. But with the about 10^{25} particles that make up everyday objects, that probability is absurdly small.

2. (40 points) You take a table tennis ball with mass 0.0027 kg, and attach it to the bottom of a tub of water with a thread. The radius of the ball is 0.020 m, and the depth of the ball is 0.26 m. The density of water is 1000 kg/m^3. The volume of a sphere is $\frac{4}{3}\pi r^3$.



(a) Find the tension in the thread.

Answer: The tension and the weight are pulling down, canceling out the buoyancy force

$$T = F_B - w = \rho g \frac{4}{3}\pi r^3 - mg = 0.30 \text{ N}$$

$\vec{T}$ is downward.

(b) You fail to use waterproof glue, and the thread comes unattached to the ball. How long does it take for the ball to rise to the surface, assuming there is no drag force on the ball?

Answer: When the thread comes unattached, the total force on the ball will be $F_B - w$ directed *upward*. Using the acceleration produced by this total force, and $\Delta y = \frac{1}{2}a(\Delta t)^2$

when it starts from rest, and Δy the depth, we solve to get

$$\Delta t = \sqrt{\frac{2\Delta y}{a}} = \sqrt{\frac{2m\Delta y}{F_B - w}} = 0.068 \text{ s}$$

This is unreasonably quick.

(c) More realistically, the drag force in the water is large, and the ball will reach terminal speed almost instantly. With $D = \frac{1}{2}C_D\rho A v^2$, where $C_D = 0.52$ and A is the cross-sectional area of the ball, calculate how long it will take the ball to rise to the surface.

Answer: At terminal speed, the drag will cancel out the other forces: $D = F_B - w$. Solving for v, we get

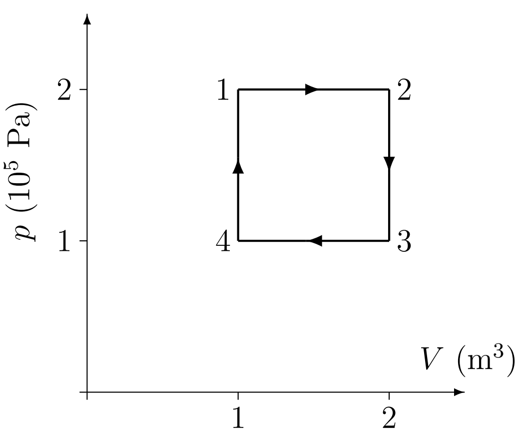
$$v_T = \sqrt{\frac{2(F_B - w)}{C_D \rho \pi r^2}} = 0.96 \text{ m/s}$$

At this constant speed, it will reach the surface in

$$\Delta t = \frac{\Delta y}{v_T} = 0.27 \text{ s}$$

Thi is much more reasonable.

3. (50 points) You have a monatomic ideal gas with an initial pressure of 2.0×10^5 Pa and volume of 1.0 m^3 (state 1). You then put it through the cycle shown in the p–V graph.



(a) For the process taking the gas from state 1 to state 2, find ΔE_{th} (the change in thermal energy), Q (the heat added to the gas), and W (the work done *by* the gas).

Answer: The work is the area under the curve,

$$W_{12} = (2 \times 10^5 \text{ Pa})\,[(2-1) \text{ m}^3] = 2 \times 10^5 \text{ J}$$

The change in thermal energy is, with $E_{th} = \frac{3}{2}NkT = \frac{3}{2}pV$,

$$\Delta E_{12} = E_2 - E_1 = \frac{3}{2}(p_2V_2 - p_1V_1) = 3 \times 10^5 \text{ J}$$

The heat put in is

$$Q_{12} = W_{12} + \Delta E_{12} = 5 \times 10^5 \text{ J}$$

(b) Find ΔE_{th}, Q, and W for the process $2 \rightarrow 3$.

Answer:

$$W_{23} = 0$$

$$\Delta E_{23} = E_3 - E_2 = \frac{3}{2}(p_3V_3 - p_2V_2) = -3 \times 10^5 \text{ J}$$

$$Q_{23} = W_{23} + \Delta E_{23} = -3 \times 10^5 \text{ J}$$

(c) Find ΔE_{th}, Q, and W for the process $3 \rightarrow 4$.

Answer: The work now is a *negative* area.

$$W_{34} = (1 \times 10^5 \text{ Pa})\left[(1-2) \text{ m}^3\right] = -1 \times 10^5 \text{ J}$$

$$\Delta E_{34} = E_4 - E_3 = \frac{3}{2}(p_4V_4 - p_3V_3) = -1.5 \times 10^5 \text{ J}$$

$$Q_{34} = W_{34} + \Delta E_{34} = -2.5 \times 10^5 \text{ J}$$

(d) Find ΔE_{th}, Q, and W for the process $4 \rightarrow 1$.

Answer:

$$W_{34} = 0$$

$$\Delta E_{41} = E_1 - E_4 = \frac{3}{2}(p_1V_1 - p_4V_4) = 1.5 \times 10^5 \text{ J}$$

$$Q_{41} = W_{41} + \Delta E_{41} = 1.5 \times 10^5 \text{ J}$$

(e) Find the *total* ΔE_{th}, Q, and W values for the full cycle. Are these as you would expect?

Answer: Adding all four steps up,

$$W_{\text{total}} = 1 \times 10^5 \text{ J}$$

$$\Delta E_{\text{total}} = 0$$

$$Q_{\text{total}} = 1 \times 10^5 \text{ J}$$

Over a full cycle, $\Delta E_{th} = Q - W = 0$, as it should be.

(f) The cycle is a heat engine. If you total the positive Q's from the steps in the cycle, you will get Q_H, the amount of heat input to the gas over a cycle. If you total the negative Q's, you will get $-Q_C$, the discarded exhaust heat. Find Q_H, Q_C and W for this cycle as a heat engine.

Answer: The positive heats are in steps $1 \to 2$ and $4 \to 1$. Therefore

$$Q_H = Q_{12} + Q_{41} = 6.5 \times 10^5 \text{ J}$$

The negative heats are in steps $2 \to 3$ and $3 \to 4$. Therefore

$$Q_C = -(Q_{23} + Q_{34}) = 5.5 \times 10^5 \text{ J}$$

The work is just the total work,

$$W = W_{\text{total}} = 1 \times 10^5 \text{ J}$$

$Q_H = W + Q_C$, as it should be.

(g) Find the efficiency of this heat engine.

Answer:

$$e = \frac{W}{Q_H} = \frac{1}{6.5} = 0.15 = 15\%$$

4. (30 points) A person with surface area of 2.0 m^2 and a skin temperature of 34°C stands naked in a room where the air is 22°C but the walls are 16°C.

(a) The thickness of the "dead air" layer next to the skin is 0.004 m, with thermal conductivity 0.025 W/m·K. What is the rate of this person's heat loss due to conduction?

Answer:

$$\frac{dQ}{dt} = \frac{kA}{L}\Delta t = \frac{(0.025)(2)}{0.004}(34 - 22) = 150 \text{ W}$$

(b) If skin's emissivity is 0.95, what is the *net* radiative heat loss to the walls? (Stefan-Boltzmann constant: 5.67×10^{-8} W/m^2·K^4.)

Answer: Convert temperatures to K first, and account for both emission and absorption:

$$\frac{dQ}{dt} = e\sigma A\left(T^4 - T_0^4\right) = (0.95)(5.67 \times 10^{-8})(2)\left(307^4 - 289^4\right) = 205 \text{ W}$$

(c) When at rest, this person has a power consumption of 100 W to keep their metabolism going. How much total power do they need from their food to stay alive?

Answer: $205 + 150 + 100 = 455$ W.

5. (30 points) As in your lab exam, you have a cart with mass m on a horizontal track. Assume friction, weight, and the normal force are the only forces acting on the cart. You let the cart go and watch it slow down. Your motion detector gives you x_i and v_i at time t_i, x_f and v_f at time t_f, and the constant acceleration a between times t_i and t_f.

(a) Write down an equation for W_f, the work done by friction in this interval.

Answer:

$$W_f = f_{\|}\Delta x = ma(x_f - x_i)$$

(b) If ΔK is the change in kinetic energy between t_i and t_f, mathematically show that $\Delta K = W_f$ or that $\Delta K \neq W_f$. (*Hint:* For 1D motion with constant acceleration, $v_f^2 = v_i^2 + 2a(x_f - x_i)$.)

Answer:

$$\Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = \frac{1}{2}m\left[2a(x_f - x_i)\right] = ma(x_f - x_i) = W_f$$

6. (30 points) You have two identical-looking cylinders, with the same mass m and radius r. Cylinder 1 has a moment of inertia $I_1 = mr^2$, while cylinder 2 has $I_2 = kmr^2$, with k an unknown constant. You let them go from rest from the exact same height on an inclined plane, and let them roll without slipping. When they reach the bottom of the incline, you measure their center-of-mass speeds, finding $v_1 = 0.94\,v_2$. What is k?

Answer: Both cylinders start with the same potential energy, and zero kinetic energy. Energy conservation means that the kinetic energies of the cylinders much be the same at the bottom of the incline. We need to account for the rotational kinetic energies, and use $v = \omega r$ for rolling without slipping.

$$\tfrac{1}{2}mv_1^2 + \tfrac{1}{2}I_1\omega_1^2 = \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}I_2\omega_2^2$$

$$\tfrac{1}{2}mv_1^2 + \tfrac{1}{2}mr^2\left(\frac{v_1}{r}\right)^2 = \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}kmr^2\left(\frac{v_2}{r}\right)^2$$

$$mv_1^2 = \tfrac{1}{2}(k+1)mv_2^2$$

$$m(0.94\,v_2)^2 = \tfrac{1}{2}(k+1)mv_2^2$$

$$(0.94)^2 = \tfrac{1}{2}(k+1)$$

Which gives

$$k = 2(0.94)^2 - 1 = 0.77$$