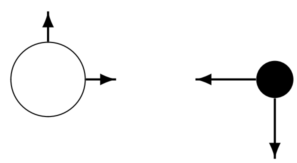
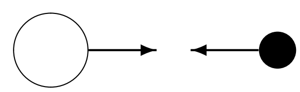
Solutions to Exam 3; Phys 185

1. (20 points) The diagrams below show a binary star system. The white star is is more massive than the one shown as a black circle, but not hugely more massive. The stars revolve around their common center of mass. On diagram (a), draw and label the velocity and acceleration vectors for each star. On (b), show the forces on each star. Assume that the stars are in deep space and we can ignore the effects of the rest of the universe on either star. Draw the sizes of your arrows such that I can tell whether v, a, and F for each star is larger, smaller, or equal to the other.

(a)



(b)



Explain your reasoning:

Answer: In a circular orbit, the acceleration and velocity will be perpendicular. Both stars rotate about their common center of mass. The acceleration and velocity of the white star is less in magnitude, due to momentum conservation, and the fact that it is more massive. The only forces are the gravitational attractions, which are equal and opposite as action-reaction pairs.

2. (40 points) You do a collision experiment with carts in the lab, but this time you work with expensive equipment that reduces friction with the track to a negligible level. You also work with carts that incorporate a spring that can be compressed and released during a collision, imparting the energy stored in the spring to the carts rebounding from the collision.

You set up the collision with a cart with mass $2m$ with initial velocity $v_{2i} = v$ heading toward a cart with mass m that starts at rest. You measure the final velocity of the cart with mass m in three different experiments, obtaining $v_{1f} = v$, $v_{1f} = \frac{4}{3}v$, and $v_{1f} = 2v$. Analyze these three experiments and determine which experiments must have had a compressed spring released during the collision.

Answer: Momentum conservation:

$$(2m)v + 0 = (2m)v_{2f} + mv_{1f} \quad \Rightarrow \quad v_{2f} = v - \frac{1}{2}v_{1f}$$

The change in total energy due to the collision will be the difference in initial and final total kinetic energies, since no relevant potential energies apply, and there is no loss to friction.

$$\Delta E = \frac{1}{2}(2m)v_{2f}^2 + \frac{1}{2}mv_{1f}^2 - \frac{1}{2}(2m)v^2$$

Putting the result from momentum conservation in there,

$$\Delta E = \frac{1}{2}(2m)\left(v - \frac{1}{2}v_{1f}\right)^2 + \frac{1}{2}mv_{1f}^2 - \frac{1}{2}(2m)v^2$$

Now, we need to investigate the sign of ΔE. If $\Delta E > 0$, extra energy has been added to the carts, which would be because of the spring being released.

The first experiment, with $v_{1f} = v$:

$$\Delta E = \frac{1}{2}(2m)\left(v - \frac{1}{2}v\right)^2 + \frac{1}{2}mv^2 - \frac{1}{2}(2m)v^2 = -\frac{1}{4}mv^2 < 0$$

This is an inelastic collision, and since $\Delta E < 0$, there's no evidence for a spring release here.

The second experiment, with $v_{1f} = \frac{4}{3}v$:

$$\Delta E = \frac{1}{2}(2m)\left(v - \frac{1}{2}\frac{4}{3}v\right)^2 + \frac{1}{2}m\left(\frac{4}{3}v\right)^2 - \frac{1}{2}(2m)v^2 = 0$$

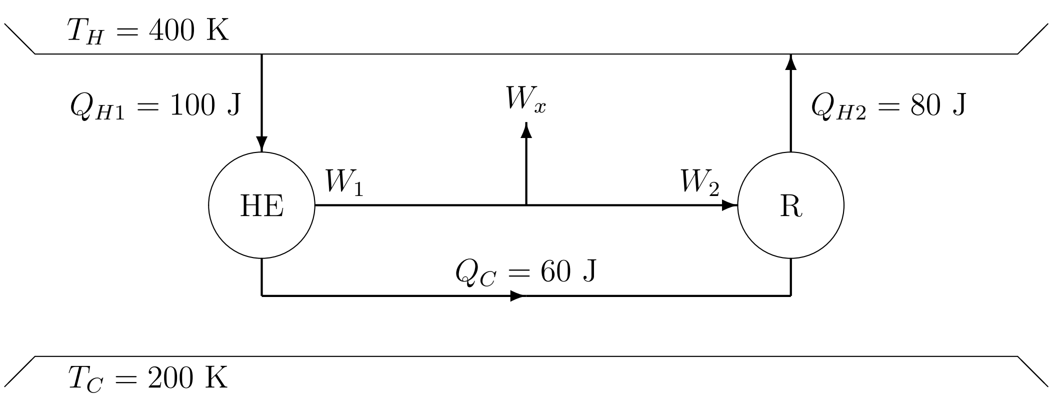
This could be an elastic collision, which could happen without a spring being released. So again, there's no evidence for a spring release.

The third experiment, with $v_{1f} = 2v$:

$$\Delta E = \frac{1}{2}(2m)\left(v - \frac{1}{2}2v\right)^2 + \frac{1}{2}m\left(2v\right)^2 - \frac{1}{2}(2m)v^2 = mv^2 > 0$$

There is no way $\Delta E > 0$ without the spring release! It must have happened in this third experiment.

3. (40 points) An inventor asks you to support his "free energy" device. He has this brilliant idea of using the exhaust heat and work from a heat engine as input to a refrigerator. He says that in each cycle, his heat engine takes in 100 J from a 400 K heat reservoir, and exhausts 60 J. But instead of just dumping that 60 J into a 200 K cold reservoir, he feeds that directly into a refrigerator. He also uses the work output by the heat engine to run the refrigerator, but since the refrigerator only returns 80 J per cycle to the hot reservoir, there is an excess work W_x that is left over for you to use as you like. You will never pay an electric bill again!



(a) Should you support this invention? Show your calculations.

Answer: From energy conservation, $W_1 = Q_{H1} - Q_C = 40$ J; $W_2 = Q_{H2} - Q_C = 20$ J; $W_x = W_1 - W_2 = 20$ J.

The maximum efficiency for a heat engine operating between these reservoirs is $e_{\max} = 1 - \frac{T_C}{T_H} = 0.5$. The actual efficiency is $e = \frac{W_1}{Q_{H1}} = 0.4 < e_{\max}$, so this part of the device is fine.

The maximum COP for a refrigerator operating between these reservoirs is $\text{COP}_{\max} = \frac{T_C}{T_H - T_C} = 1$. The actual COP $= \frac{Q_C}{W_2} = 3 > \text{COP}_{\max}$. So the refrigerator component of this device is impossible.

(b) If in (a), you found that this invention was workable, are there any values of Q_C and T_C for which the invention wouldn't work? If your answer to (a) was "no," are there any values of Q_C and T_C for which the invention would work? Explain.

Answer: No matter what Q_C and T_C are, this device extracts a net heat of $Q_{H1} - Q_{H2} = 20$ J from the hot reservoir, converting it into $W_x = 20$ J of work without any exhaust heat. Such a free conversion of heat into work is impossible—it violates the second law of thermodynamics.

4. (30 points) You have a lake which is frozen, with a layer of ice of 0.11 m thickness separating water at 0.0°C from the outside air at −5.0°C. Assuming that the only relevant heat conduction mechanism is conduction, what is $\Delta L/\Delta t$, the rate at which the thickness of the ice increases? You can find all the physical data you require about ice and water in your textbook, in tables in chapter 12 and the back flap.

Answer: We will need the thermal conductivity of ice, $k = 1.7$ W/m·K; the density of ice, $\rho = 920$ kg/m^3; and the latent heat of fusion for water freezing into ice, $L_f = 3.33 \times 10^5$ J/kg.

The rate of heat transfer from the water to the air is

$$\frac{Q}{\Delta t} = k\frac{A}{L}\Delta T$$

where A is the surface area of the lake. This lost heat Q goes into freezing the water at 0.0°C, so that $Q = mL_f$. The mass frozen is $m = \rho V = \rho A \Delta L$, where ΔL is the thickness added to the ice layer. Therefore

$$\frac{L_f \rho A \Delta L}{\Delta t} = k\frac{A}{L}\Delta T \quad \Rightarrow \quad \frac{\Delta L}{\Delta t} = \frac{k\Delta T}{\rho L L_f} = 2.5 \times 10^{-7} \text{ m/s}$$

This is quite slow; once a thick layer of ice forms, it is a pretty good insulator.

5. (40 points) If an astronaut were to step out into the vacuum of empty space, far away from all stars and other sorts of heat, she would be in an environment close to absolute zero:

the 3 K of the cosmic microwave background radiation, in fact. To calculate if she would freeze to death, you gather the following information. The thermal conductivity of her clothes: 0.05 W/m·K. Thickness of her clothes: 0.010 m. Her surface area: 1.8 m^2. The emissivity of her clothes surface: 0.5. Stefan-Boltzmann constant: 5.67×10^{-8} W/m^2·K^4. The surface temperature of her clothes: 35°C. The specific heat of mammalian tissue: 3400 J/kg·K. Her mass: 56.0 kg. Rate of thermal energy generation by a human body at rest: 100 W. (Not all these values are necessarily useful!)

(a) Calculate the rate the astronaut absorbs heat from the microwave background.

Answer: With $T_\mu = 3$ K,

$$\frac{Q}{\Delta t} = e\sigma A T_\mu^4 = 4.13 \times 10^{-6} \text{ W}$$

This is miniscule, as you might expect.

(b) Calculate her *net* rate of heat loss.

Answer: In outer space, you won't lose any heat due to conduction or convection, so the radiative heat loss is all you need to consider. Now, $T = 35 + 273 = 308$ K, so

$$\frac{Q}{\Delta t} = e\sigma A T^4 = 459 \text{ W}$$

Technically, you should subtract the absorption due to the cosmic microwave background, but that is negligible in comparison. What is not negligible is the 100 W generated by your metabolism, so

$$\frac{Q_{\text{net}}}{\Delta t} = 459 \text{ W} - 100 \text{ W} = 359 \text{ W}$$

This will turn out not to be too bad.

(c) Calculate the rate at which her body temperature will drop, which is $\Delta T/\Delta t$.

Answer: Using $Q = mc\Delta T$ for temperature change,

$$359 \text{ W} = \frac{mc\Delta T}{\Delta t}$$

and so

$$\frac{\Delta T}{\Delta t} = \frac{359 \text{ W}}{(56.0 \text{ kg})(3400 \text{ J/kg·K})} = 1.89 \times 10^{-3} \text{ K/s}$$

This is slow.

(d) How long will it take for her to cool by 1 K?

Answer:

$$\frac{1 \text{ K}}{1.89 \times 10^{-3} \text{ K/s}} = 530 \text{ s} \approx 9 \text{ minutes}$$

Freezing to death is not her immediate problem in this situation.

6. (30 points) Take a small bubble of air at a depth d below the ocean surface. There are n moles of air in the bubble, and air is approximated very well as an ideal gas. Let's assume that the bubble is small enough that we can assume a single depth and a single pressure value accurately characterizes the bubble. Let's also assume that the ocean has a constant temperature T at any depth, and that the air is always in thermal equilibrium with the ocean. Use p_{atm} to represent atmospheric pressure and ρ_w to represent the density of water.

(a) Write down an equation for V, the volume of the bubble.

Answer: Use $pV = nRT$, and the fact that the pressure within the bubble will be equal to the water pressure at its depth, $p = p_{atm} + \rho_w g d$:

$$V = \frac{nRT}{p_{atm} + \rho_w g d}$$

(b) Now write down an equation for the buoyancy force F_B on the bubble.

Answer: The buoyancy force is equal to the weight of water of an equal volume:

$$F_B = \rho_w V g = \frac{nRT\rho_w g}{p_{atm} + \rho_w g d}$$

(c) Make a rough sketch of the buoyancy force versus depth. Make sure the sketch is clear about whether F_B almost at the surface ($d = 0$) is zero, infinite, or a finite value.

Answer: The curve for F_B should begin with a finite value for $d = 0$, and monotonically decrease without ever becoming zero.