

**Note:** As I walk around, you can ask me for help; for example, to supply an equation or a number you have forgotten down, or to give you algebra aid. If you do, however, I will write down what help I provided on your exam, and grade your answer accordingly.

**1. (40 points)** A layer of material 1 with index of refraction  $n_1$  stands on a horizontal slab of material 2 with index of refraction  $n_2$ , with  $n_2 > n_1$ . Light from within the slab of material 2 is incident on the boundary between 1 and 2. Find an equation for the maximum angle of incidence for which a light ray can emerge into the air above layer 1.

air

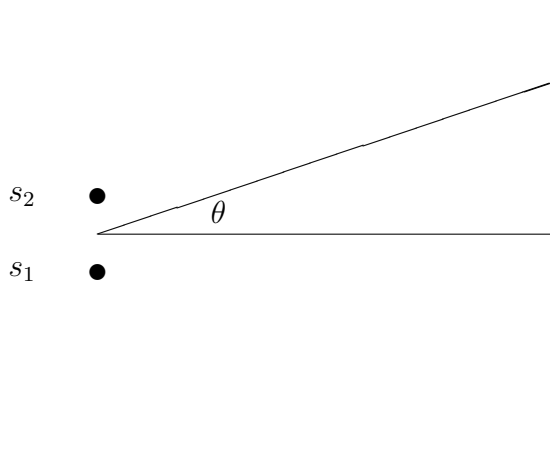


$n_1$

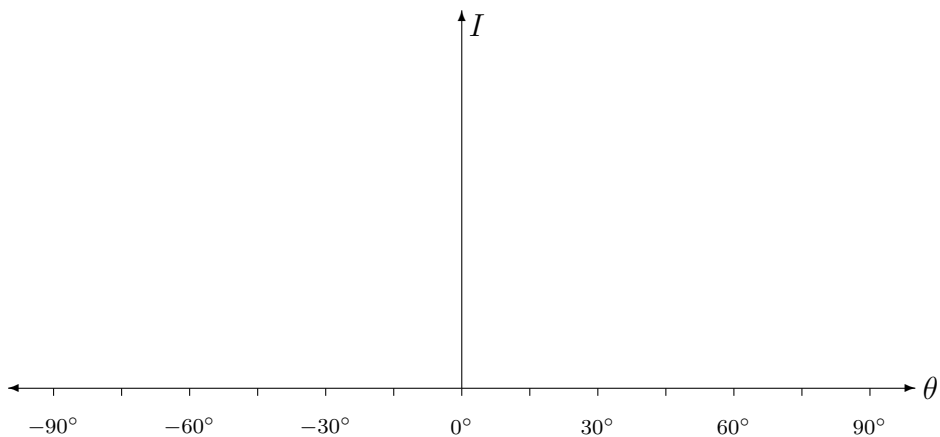


$n_2$

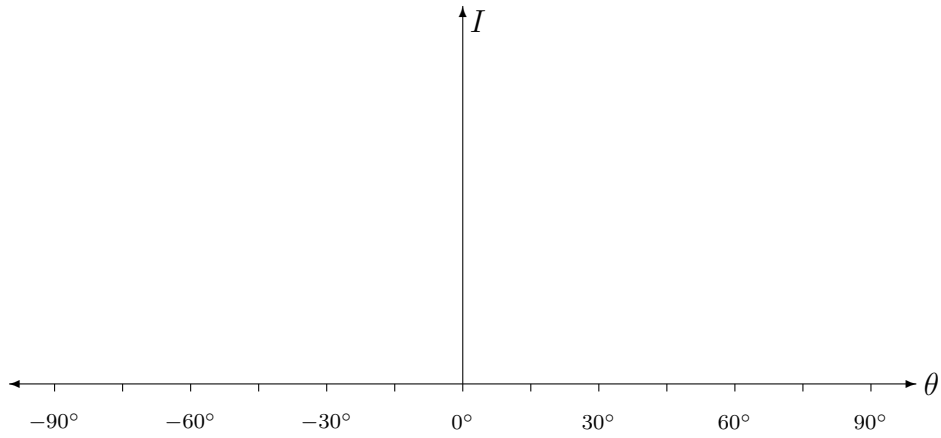
2. (40 points) You have two sources of waves with identical frequencies, amplitudes, and wavelengths,  $s_1$  and  $s_2$ , separated by a distance of 2.2 wavelengths:  $d = 2.2\lambda$ . To the right there is a screen that records intensity at angles  $\theta$ .



- (a)  $s_1$  and  $s_2$  are completely in phase: they emit peaks and troughs of waves simultaneously. Plot a rough intensity versus angle graph. Be careful about whether your minima are exactly zero or not. Calculate and indicate the angles at which the *maxima* occur.



- (b)  $s_1$  and  $s_2$  are completely out of phase: when one emits a peak the other emits a trough. Plot a rough intensity versus angle graph. Be careful about whether your minima are exactly zero or not. Indicate the angles at which the *minima* occur.



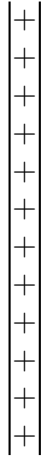
**3. (30 points)** You have a spring with stiffness (spring constant)  $k$  with a mass  $m$  attached. At time  $t = 0$ , you extend the spring by a distance  $d$  away from equilibrium, and let it go from rest. At the same time, you also have another spring with stiffness  $2k$  with a mass of  $m/2$  attached. At  $t = 0$ , you extend this second spring by  $2d$  away from equilibrium, and let it go from rest.

On the graph below, sketch the position of the mass versus time for both springs, over *two periods* of the first spring. Show your calculations below the graph.

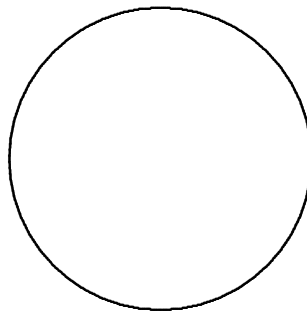


4. (40 points) Roughly sketch equipotential lines (solid lines) and electric field lines (dashed lines with arrows indicating direction) in the following situations.

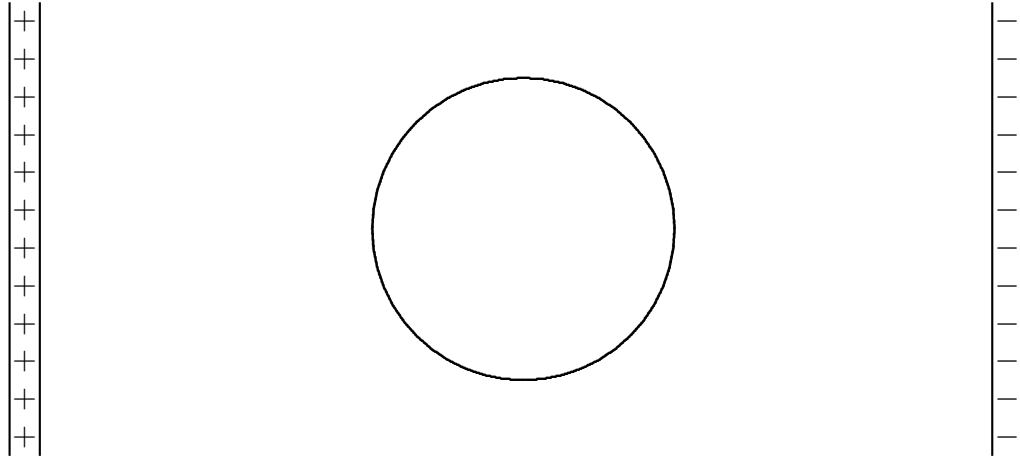
(a) Two parallel plates with equal and opposite charges upon them.



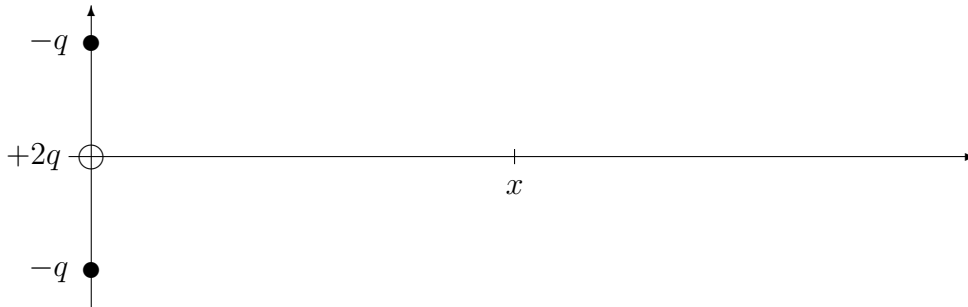
(b) Now say you placed a metal cylinder in between the plates. Inside the cylinder, roughly indicate how charges will be redistributed by placing + and - signs. Outside, roughly draw in the equipotential lines and electric field lines due to this induced charge distribution.



- (c) Now draw in the equipotential lines and total electric field lines for the combined configuration: what is due to the plates *plus* the induced charge distribution on your cylinder. Include the lines within the cylinder. This is what you would see in the lab with your voltage probe.



5. (50 points) You have a quadrupole charge distribution with charges  $-q$  at  $x = 0, y = a$  and  $x = 0, y = -a$ , and a charge  $+2q$  at  $x = 0, y = 0$ . The positions of the charges relative to each other are fixed.



- (a) Find the total electric field due to the quadrupole, at a point a distance  $x$  from the origin on the  $x$  axis. In other words, find the total  $E_x$  and  $E_y$  as functions of  $k$ ,  $q$ ,  $a$ , and  $x$ .

- (b) If you can cast your previous answer in a form that puts all the  $a$ -dependence in a term that goes like  $(1 + a^2/x^2)^{-3/2}$ , you can use the approximation

$$\left(1 + \frac{a^2}{x^2}\right)^{-3/2} \approx 1 - \frac{3a^2}{2x^2}$$

valid for  $a \ll x$ . Use this to show that for large  $x$  values, the electric field due to a quadrupole goes like  $E \propto x^{-4}$ , an inverse fourth power law.