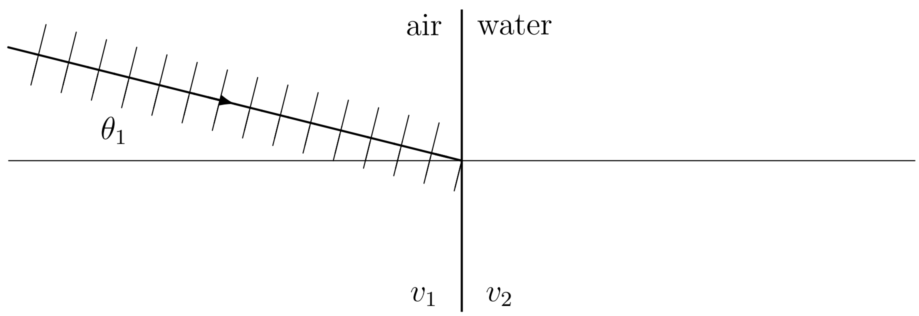
# Solutions to Exam 1; Phys 186

**1. (30 points)** You have a sound wave traveling from air into water.

(a) Complete the following diagram on the water side. The parallel lines indicate wavefronts, while the line with an arrow is the direction of energy transfer. The speed of sound in air is $v_1$, the speed of sound in water is $v_2$, and $v_2 > v_1$.




**Answer:** Since $f$ doesn't change, and $v = \lambda f$, $\lambda_2 > \lambda_1$. The wavefronts on the water side must be longer, which means that the direction of energy transfer in water must bend closer to the interface, so that $\theta_2 > \theta_1$.

(b) What is the equation that gives the relationship between $v_1$, $\theta_1$, $v_2$, and $\theta_2$? Explain your reasoning.

**Answer:** With light, you have $n_1 \sin\theta_1 = n_2 \sin\theta_2$, which is

$$\frac{c}{v_1}\sin\theta_1 = \frac{c}{v_2}\sin\theta_2$$

The $c$ does not play any vital role here, and it cancels out. The remaining relationship,

$$\frac{1}{v_1}\sin\theta_1 = \frac{1}{v_2}\sin\theta_2$$

is correct for *any* sort of wave, not just light or sound.

**2. (40 points)** Imagine that in the lab you had sources of "Q-rays." You also have detectors indicating the intensity of incident Q-rays. There are two theories about Q-rays: (1) the source emits a stream of small, invisible, and extremely numerous Q-particles; (2) the source emits Q-waves.

You can test whether there are Q-particles by observing that particles can collide with each other, while waves just interfere. You set up two Q-beams whose paths cross, and see if there is any sideways scattering from the beam paths due to particles colliding. If so, that is strong evidence for particles. If, however, there is no scattering, it doesn't guarantee waves. After all, the particle size $d$ might be very small, making collisions unlikely.

(a) Describe an experiment in which you can test for Q-waves. Explain its rationale: tell me what a positive result would be and why it would be strong evidence for waves.

**Answer:** The signature of waves is interference. If you pass the Q-rays from the source through a double-slit or diffraction grating experiment, and you see the classic multiple intensity peaks behaving according to $\sin\theta = m\lambda/d$, you will have a result that screams "waves!"

(b) Would a negative result in your experiment guarantee particles, or would some kinds of waves also give a negative result? Explain.

**Answer:** No. There are many ways in which you can get an ambiguous result. For example, if $\lambda \ll d$, you will get a double-peak intensity pattern for a double-slit experiment, exactly the same as for particles.

**3. (40 points)** Imagine a single-frequency electromagnetic wave (sine wave) going through an area in which there is an electron with initial position $x_e = y_e = 0$. Consider only the electric field: it points up and down (depending on what part of the sine wave is going through) along the $y$-axis, while the wave travels in the $+x$-direction. At all times $E_x = 0$ and $E_y$ is the same at all points along the $y$-axis. At time $t = 0$, at all points where $x = 0$, $E_y = 0$, and after a very small time $\epsilon$ passes, the positive part of the electric field moves in, so that $E_y > 0$ when $t = \epsilon$. The frequency of the electric wave is $f$, its speed is $c$.

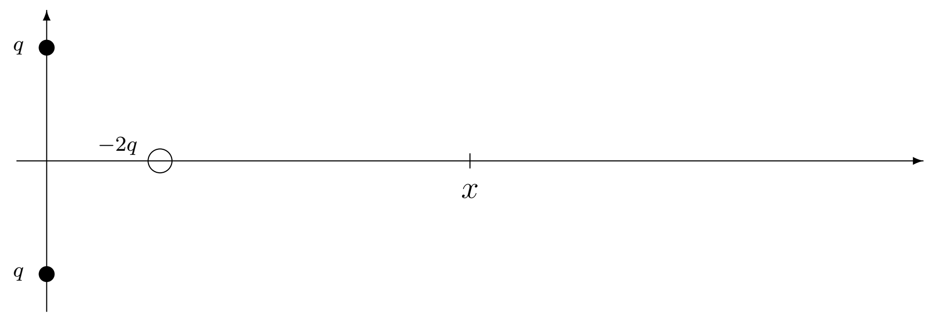
Make a qualitative graph of the electric force experienced by the electron, as a function of time $t$. Then make a qualitative graph of $x_e$ and $y_e$ vs. $t$. Explain your reasoning.

**Answer:** The electron has a negative charge, so the force on it opposes the electric field. The force graph will therefore be an inverted sine curve, where at $t = \epsilon$, $F_E < 0$. Now, this force is entirely in the $y$-direction, so there will be no acceleration in the $x$-direction. Therefore the $x_e$ graph will just be a straight line—any straight line. For $y_e$, all you have to do is recognize that the force graph is just the same shape as you would get for a mass oscillating on the end of a spring. Therefore the position graph will also be a sine curve, inverted in relation to the force curve. If the electron had an initial velocity $v_y \neq 0$, then you will have a sine curve added to a sloping straight line.

**4. (30 points)** Say you set up two finite metal plates and impose a 4V voltage difference between them, much like you did in your equipotential lines lab. For the following two plate arrangements, make qualitative drawings of the 0V, 1V, 2V, 3V, and 4V equipotential lines. Then add the electric field lines. Be sure to show what happens *outside* the plates.

**Answer:** For the large but finite capacitor, you will get edge effects: the equipotential lines will curve back slightly toward the the plates, and circle around the back of the plates, where they will be very widely separated. With the second capacitor, where the plate size is smaller than the separation, what you have is very similar to a dipole.

**5. (60 points)** You have a charge distribution with point charges $q$ at $x = 0, y = d$ and $x = 0, y = -d$, and a charge $-2q$ at $x = d, y = 0$. The positions of the charges relative to each other are fixed.




(a) Find the total electric field at a point a distance $x$ from the origin on the $x$ axis. In other words, find the total $E_x$ and $E_y$ as functions of $k$, $q$, $d$, and $x$.

**Answer:** If you draw your $\vec{E}$ arrows for each charge correctly, you can see that the $y$-components will cancel each other out. So let's worry about the $x$-components only.

The $+q$ charges will each contribute

$$E_{+x} = \frac{kq}{r^2}\cos\theta$$

where $r^2 = x^2 + d^2$ and $\cos\theta = x/r$. The $-2q$ charge will contribute

$$E_{-x} = -\frac{2kq}{(x-d)^2}$$

Adding everything up,

$$E_{Tx} = 2kq\left[\frac{x}{(x^2+d^2)^{3/2}} - \frac{1}{(x-d)^2}\right]$$

(b) Find the total electric potential at the same point.

**Answer:** We need not worry about sines and cosines for voltages. So we add everything together:

$$V_T = \frac{kq}{r} + \frac{kq}{r} - \frac{2kq}{x-d} = 2kq\left[\frac{1}{\sqrt{x^2+d^2}} - \frac{1}{x-d}\right]$$

(c) In class, we found that a dipole's electric field magnitude behaved like $E \approx km/x^3$ for $x \gg d$, where $m$ was a constant. Find the electric field for this charge distribution when $x \gg d$, and show that $E \approx km/x^3$ in this case as well. Find $m$ in terms of $q$ and $d$ for this charge arrangement.

You may use $(1+\epsilon)^p \approx 1 + p\epsilon$ for $\epsilon \ll 1$. Also note that when $x \gg d$, $d/x \gg (d/x)^2$, which means you can neglect terms that go like $1/x^4$ when compared to terms that go like $1/x^3$. I'm available for algebra help.

**Answer:** write the answer from (a) as:

$$\begin{aligned} E_{Tx} &= 2kq\left[x\,x^{-3}\left(1+\frac{d^2}{x^2}\right)^{-3/2} - x^{-2}\left(1-\frac{d}{x}\right)^{-2}\right] \\ &\approx 2kqx^{-2}\left[\left(1-\frac{3d^2}{2x^2}\right) - \left(1+\frac{2d}{x}\right)\right] \\ &\approx -\frac{4kqd}{x^3} \end{aligned}$$

Therefore $m = -4qd$.