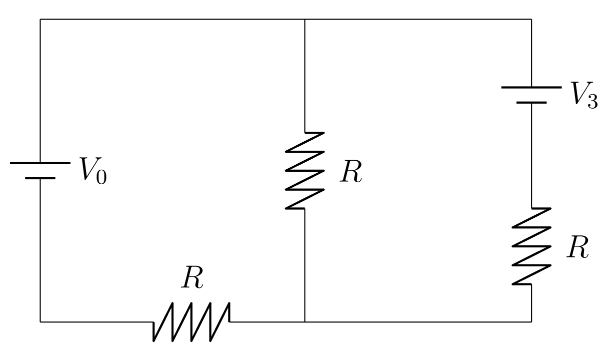
Solutions to Exam 2; Phys 186

1. (30 points) A friend of yours hears that "space and time are curved" on a science fiction show, and decides to ask you to explain that statement, since you're taking a physics course. How would you explain it? What examples would you use?

Answer: Answers will differ. But I would expect you would say something about how the time dilation factor γ is zero for a photon, which means that a photon always takes the shortest path between two locations, covering it in zero time in its own frame. Then you could point out that accelerating frames are indistinguishable from frame with gravity without outside information. The path of light curves in an accelerating frame, and therefore also in a frame with gravity. But this must mean, then, that in the presence of gravity, the shortest path is curved! This can only happen with a curved underlying space (and time, since time is also affected by relativity). You can use the analogy of curved "great circles" as shortest paths on the surface of a sphere. In any case, mass/energy curves space and time, which is the same as gravity, which tells mass/energy how to move.

2. (40 points) You have the following circuit with three identical resistances R and two batteries with voltage V_0 and V_3.



(a) Solve for I_3, the current through the battery with V_3, in terms of R, V_0, and V_3.

Answer: Call the current from the battery I_1, which then splits into

I_2 and I_3. The junction equation is then:

$$I_1 = I_2 + I_3$$

The loop equations:

$$V_0 = I_1 R + I_2 R \qquad I_2 R = V_3 + I_3 R$$

Solving these, we get

$$I_3 = \frac{V_0 - 2V_3}{3R}$$

(b) You can use a second battery like V_3 in this circuit to control the power dissipated by the resistor in series with V_3. Say you knew the power dissipated by this resistor had to be $V_0^2/36R$. What would V_3 then have to be?

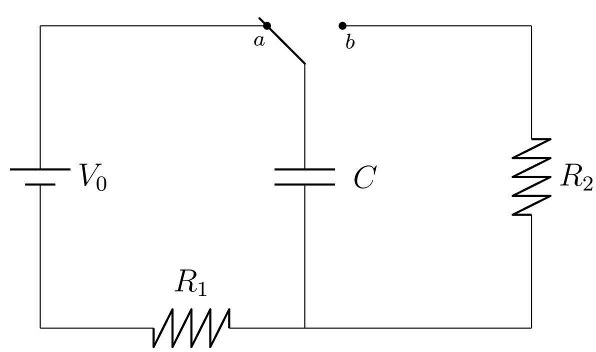
Answer: The power dissipated by a resistor is $P = VI = I^2 R$. Using the result from (a) for I_3, this means

$$\left(\frac{V_0 - 2V_3}{3R}\right)^2 R = \frac{V_0^2}{36R}$$

This is satisfied when

$$V_3 = \frac{V_0}{4}$$

3. (40 points) Here is a simplified (oversimplified) model of a circuit for a camera flash. The resistance R_1 is considerably larger than R_2. When the switch is at a, the capacitor C slowly recharges. When the switch is at b, C rapidly discharges.



(a) Say the switch remains at a for a long time in order to fully charge up the capacitor. This is a "long time" compared to what?

Answer: The time save for charging up is R_1C—so the time must be long compared to R_1C.

(b) What is the power dissipated by R_2 immediately after the switch is flipped to b? Explain, using this, why a flash requires a small value for R_2.

Answer: Since the capacitor was fully charged, the voltage across it immediately after the switch is flipped will be V_0. (It would not have had any time to discharge yet.)Therefore, using a lop equation, the voltage across the resistor will also be V_0 and the current going through will be V_0/R_2. The power is then

$$P = \frac{V_0^2}{R_2}$$

A flash requires a large burst of energy delivered in a short amount of time. Therefore P should be large—which is why R_2 should be small.

(c) Say $C = 12\ \mu\text{F}$, and $R_2 = 0.21\ \Omega$. How long will it take for the capacitor to discharge 90% of its starting charge?

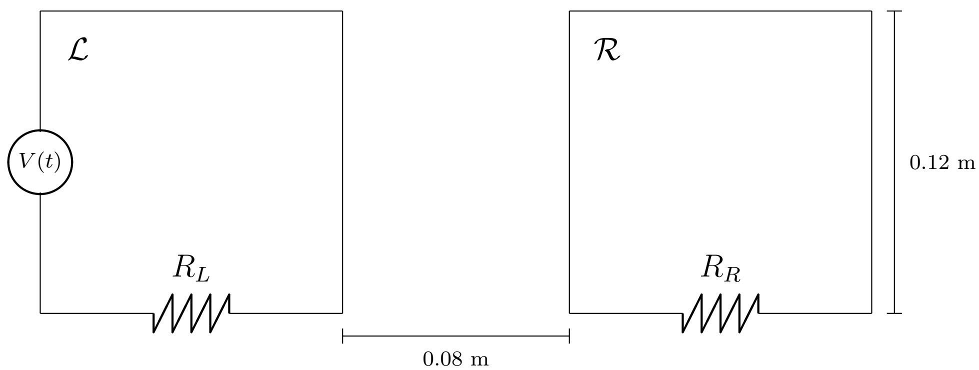
Answer: The capacitor needs to go down to $1 - 0.9 = 01$ of its original charge. Uisng the exponential discharge relationship,

$$Q = Q_0 e^{-t/R_2C} \qquad \Rightarrow \qquad \frac{Q}{Q_0} = 0.1 = e^{-t/R_2C}$$

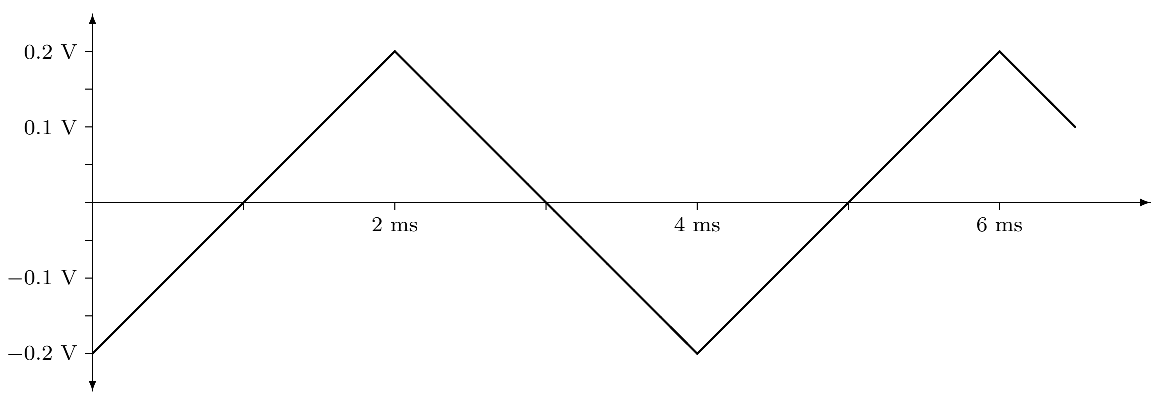
Therefore

$$t = -R_2C \ln 0.1 = 5.8 \times 10^{-6}\ \text{s}$$

4. (50 points) You have two circuits next to each other:

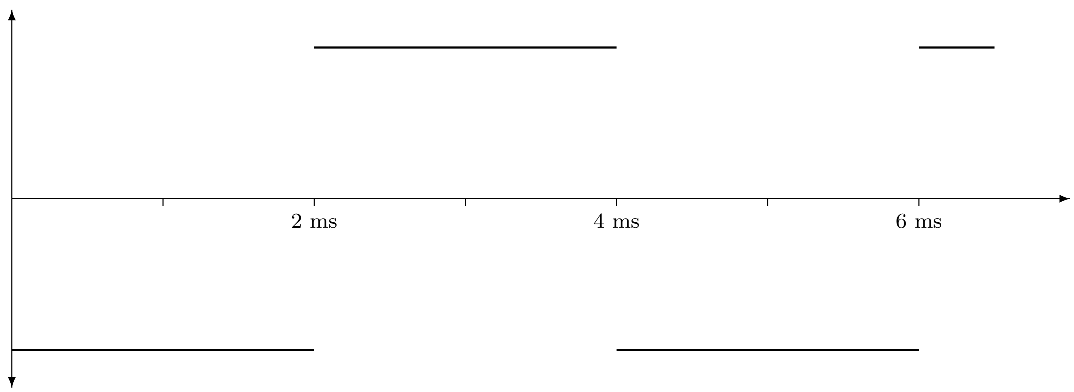


Each circuit is square with 0.12 m sides, and 0.080 m separates the right edge of the left circuit ($\mathcal{L}$) from the left edge of the right circuit ($\mathcal{R}$). Both have the same resistance values $R_L = R_R = 0.50\ \Omega$, and $\mathcal{L}$ has a voltage source which produces a sawtooth waveform $V(t)$, which looks like the following on an oscilloscope:



(a) Sketch the shape of the waveform you will see if you measure the voltage across R_R with an oscilloscope. Don't put in any voltage numbers—just sketch the waveform.

Answer: The induced voltage $V \propto -\frac{d}{dt}\Phi \propto -\frac{d}{dt}B \propto -\frac{d}{dt}I$. Therefore the shape of the waveform will go like the slope of the $V(t)$ curve:



(b) Now let's make some approximations to estimate the amplitude of the voltage waveform induced in $\mathcal{R}$. There are four wire segments in $\mathcal{L}$: the left, top, right, and bottom on the diagram. The current in each wire produces a magnetic field through $\mathcal{R}$. Only one of the following makes the largest contribution to the magnetic flux through $\mathcal{R}$—we will just take that and ignore the rest. Circle your answer:

<u>The right wire</u> The top and bottom wires The left wire

Brief explanation: The magnetic field falls off with distance; the right wire is closest to $\mathcal{R}$ and therefore will give the largest contribution.

The magnetic field produced by the wire segment you picked will not be uniform through $\mathcal{R}$. But we are looking for a rough estimate, so we will assume that is is uniform. The magnitude of B at what part of $\mathcal{R}$ will be a representative value to use in this uniform approximation?

The right edge <u>The center</u> The left edge

Brief explanation: The magnetic field falls off with distance; the center is therefore a more representative value between the right and left extremes.

Now we need an equation that will help us get the magnetic flux:

Loop: $B = \dfrac{\mu_0 I}{2r}$ <u>Long wire:</u> $B = \dfrac{\mu_0 I}{2\pi r}$ Wire: $F = ILB$

Brief explanation: The loop equation refers to the field at the *center* of a circular loop, which is not at all this situation. The force on a wire is totally irrelevant. Though the magnetic field created by a long wire is not a very accurate approximation here, since the wire length is comparable to the distance to the wire, we only need a rough estimate.

Finally, use all this and estimate the *amplitude* of the waveform sketched in part (a).

Answer: We use $r = 0.14$ m as the distance to the center of $\mathcal{R}$, and the area $A = (0.12\ \text{m})^2$. In that case

$$V = -\frac{d}{dt}\Phi = -\frac{\mu_0 A}{2\pi r}\frac{d}{dt}I$$

The current is $I = V(t)/R_L$, therefore

$$\frac{d}{dt}I = \frac{1}{R_L}\frac{d}{dt}V(t)$$

The rate of change of the sawtooth voltage is the slope, ± 200 V/s, depending on whether it is increasing or decreasing. The induced voltage will therefore be

$$V = \mp\frac{\mu_0 A}{2\pi r R_L}\frac{d}{dt}V = \mp 8.2 \times 10^{-6}\ \text{V}$$

So the amplitude is 8.2×10^{-6} V.

5. (40 points) You have a proton and an antiproton at rest on Earth. They annihilate to produce a muon-antimuon pair: $p + \bar{p} \to \mu^- + \mu^+$. The muon heads toward the Moon, 3.8×10^8 m away, and the antimuon is captured by a detector here on Earth. The typical lifetime of a muon is 2.2×10^{-6} s. Will the muon make it to the Moon to be captured by a detector there? A muon's mass is $m_\mu = 1.9 \times 10^{-28}$ kg, or 110 MeV/c^2. A proton's mass is $m_p = 1.7 \times 10^{-27}$ kg or 940 MeV/c^2. The speed of light is 3.0×10^8 m/s. Note:

- Relativistic energy (γmc^2) and momentum ($\gamma m\vec{v}$) are both conserved in this reaction. Show how you use both.

- You'll get a **bonus +5 points** if you solve this using the masses given in MeV/c^2.

Answer: The masses of particles and their antiparticles are identical. Therefore, with the proton and antiproton starting from rest, momentum conservation looks like

$$0 = \gamma_{\mu^-} m_\mu \vec{v}_{\mu^-} + \gamma_{\mu^+} m_\mu \vec{v}_{\mu^+}$$

This implies the velocities of the muon and antimuon are equal and opposite, and that therefore $\gamma_{\mu^-} = \gamma_{\mu^+}$.

Energy conservation then gives, with $\gamma_p = \gamma_{\bar{p}} = 1$ because they are at rest,

$$2m_p c^2 = 2\gamma_\mu m_\mu c^2 \qquad \Rightarrow \qquad \gamma_\mu = \frac{m_p}{m_\mu} = 8.6$$

This is a large γ, which implies the speed of the muon $v \approx c$. In the frame of reference of an observer on Earth, the muon's lifetime will increase due to time dilation; it will live for $\Delta t = \gamma_\mu (2.2 \times 10^{-6}\ \mathrm{s})$. During this time it will travel close to the speed of light, covering

$$L_0 = c\gamma_\mu (2.2 \times 10^{-6}\ \mathrm{s}) = 5.6 \times 10^3\ \mathrm{m} < 3.8 \times 10^8\ \mathrm{m}$$

The muon will fall far short of the Moon.