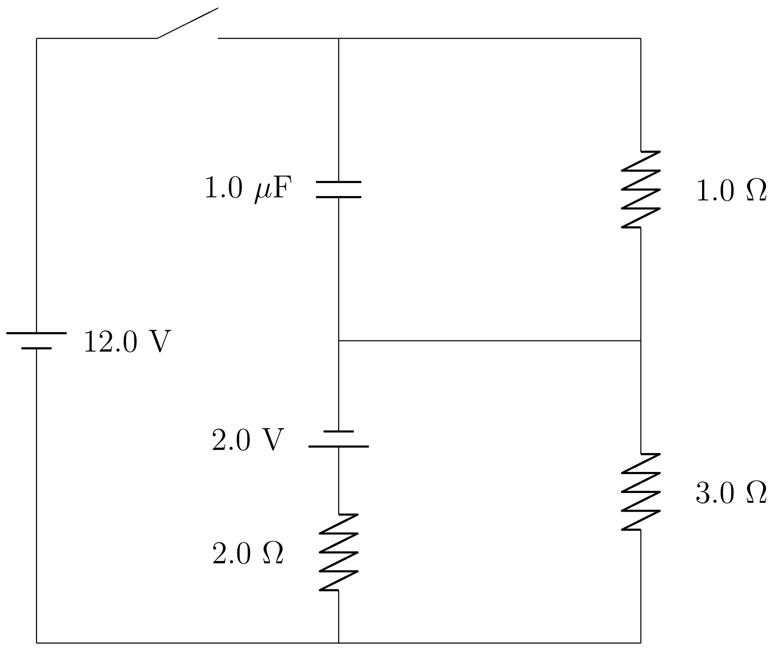
Solutions to Exam 2; Phys 186

1. (70 points) Just before the switch is closed, the capacitor in the following circuit is completely discharged. You then close the switch.



(a) Find the currents through all the resistors *immediately after* the switch is closed, before the capacitor has any time to charge up at all.

Answer: If the capacitor has had no time to build up any charge, the voltage across it must be $V = Q/C = 0$. The 1 Ω resistor is in parallel with the capacitor, hence it sees the same voltage of 0. Therefore no current flows through this resistor—all the current will go through the capacitor, bypassing this resistor. $I_1 = 0$. In other words, an uncharged capacitor behaves like a closed switch, shorting out anything in parallel to it.

This simplifies the circuit: in effect, we can replace the capacitor with a straight wire, and remove the 1 Ω resistor from the circuit. We now analyze the circuit with the two remaining resistors. The junction equation is

$$I_0 = I_2 + I_3$$

where I_0 is the current put out by the 12 V battery. The two loop equations are

$$12\text{ V} + 2\text{ V} = (2\ \Omega)I_2$$

$$(2\ \Omega)I_2 = 2\text{ V} + (3\ \Omega)I_3$$

The first loop equation directly gives $I_2 = (12+2)/2 = 7$ A. Putting that into the second loop equation produces $I_3 = (2 \cdot 7 - 2)/3 = 4$ A.

(b) Find the currents through all the resistors *a very long time after* the switch is closed, after the capacitor has completely charged up.

Answer: After a long time, the capacitor will have fully charged, and the current through it will be zero. All the current will go through the parallel 1 Ω resistor. In other words, in this case we remove the capacitor, replacing it with an open switch. The circuit equations are very similar, with $I_0 = I_1$ now going through the 1 Ω resistor as well.

$$I_1 = I_2 + I_3$$

$$12\text{ V} + 2\text{ V} = (2\ \Omega)I_2 + (1\ \Omega)I_1$$

$$(2\ \Omega)I_2 = 2\text{ V} + (3\ \Omega)I_3$$

Solving, we end up with $I_2 = 4$ A, $I_3 = 2$ A, and $I_1 = 6$ A.

(c) Sketch qualitative I vs t graphs for each of the currents. The switch closes at $t = 0$.

Answer: I_1 will rise from 0 and level off asymptotically at 6 A. I_2 and I_3 will have exponential decay curves between their $t = 0$ and $t \to \infty$ values.

2. (50 points) This question is about charges in electric and magnetic fields.

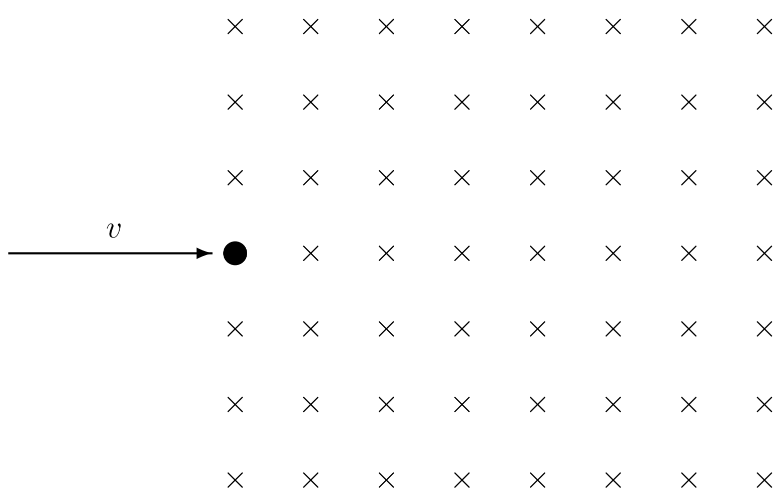
(a) Magnetic fields act on moving charges. To get electrons moving at high speed, we can accelerate them by using a high voltage. Say an electron with charge $-e$ and mass m starts at rest, and gains kinetic energy by accelerating through a voltage difference of $-V_a$. What is v, its final speed, in terms of e, m, and V_a?

Answer: The energy gained by the electron will be eV_a. This goes into kinetic energy, $\frac{1}{2}mv^2$. Solving,

$$v = \sqrt{2eV_a/m}$$

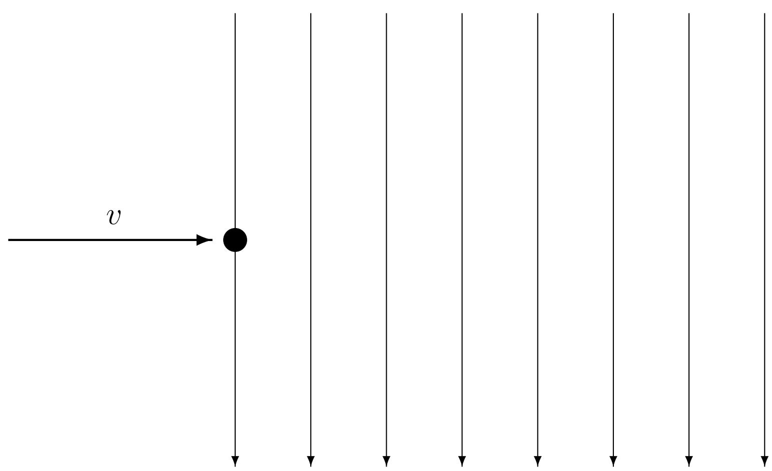
(b) An electron with speed v, moving toward the right, enters a region with a uniform magnetic field with magnitude B. The magnetic field points into the page. What is the magnitude of the magnetic force on the electron, F_B, in terms of B, e, m, and V_a? Draw its direction as the rightward-moving electron just enters the region with the uniform magnetic field into the page.

(c) Draw a diagram showing the *trajectory* of the electron in the same magnetic field. (The black dot is, once again, the electron entering the magnetic field.)



Answer: This magnetic field will produce a force $F_B = evB$, always perpendicular to the velocity. This bends the electron in a circular path. By the right-hand-rule, and the fact that the electron has a negative charge, $\vec{F}_B$ will point downward on the page. Therefore the electron bends downward in a circular arc.

(d) Now say that the electron, instead of emerging into a magnetic field, enters a region with a uniform electric field. The electron with speed v, moving toward the right, comes in between the plates of a parallel plate capacitor that produces an electric field with magnitude E. The field points downward on your page. Draw a diagram showing the trajectory of the electron in the uniform electric field.



Answer: The electric force is $F_E = eE$ in magnitude. Since the electron charge is negative, $\vec{F}_E$ points upward on the page. This *constant* force will bend the electron upwards, in a parabolic trajectory.

(e) The capacitor has a voltage difference of V_c across its plates, which are separated by a distance d. What is E, the magnitude of the electric field in this capacitor, in terms of V_c and d? **Answer:** The voltage in a capacitor increases linearly with distance, from 0 to V_c. Therefore

$$E = \frac{V_c}{d}$$

(There are multiple ways to get this.)

(f) Now say that the electron with speed v emerges into an region where *both* a uniform magnetic field and a uniform electric field exists in the ways described in (b) and (d). If the magnitude B is just right, the electric force and magnetic force will cancel each other out and the electron will not be affected. Find this value of B in terms of V_c, V_a, e, m, and d.

Answer: Setting $F_E = F_B$,

$$evB = e\frac{V_c}{d} \qquad \Rightarrow \qquad B = \frac{V_c}{vd} = \frac{V_c}{d\sqrt{2eV_a/m}}$$

3. (40 points) You run into someone selling a "special antimagnetic covering" for your phone, claiming that it is at risk from "fluctuations in the Earth's magnetic field." In fact, they say, the Earth's magnetic field may flip direction any day now, and you'll end up with magnetic North changing to magnetic South in the space of an hour. When that happens, your phone circuits will be disrupted and destroyed, and so you need protection.

You know no such thing is about to happen, but just in case, do a quick calculation to estimate whether it might be worth buying protection from a magnetic field flip. You can find any numbers you need in your textbook, or make up what you think are reasonable numbers for your phone or anything else you need.

Answer: Calculations will differ. But you want to estimate the rate of change of magnetic flux through the phone circuits. Say it takes 1 hour, 3600 seconds, for the magnetic field to go from $+B$ to $-B$, where $B \approx 5 \times 10^{-5}$ T, a rough value for the Earth's magnetic field. Therefore

$$\frac{dB}{dt} \approx \frac{B - (-B)}{3600} = 2.8 \times 10^{-8}\ \text{T/s}$$

Say your phone has circuits of about 5 cm on a side. So $A \approx 2.5 \times 10^{-3}$ m^2. Therefore you induce

$$V = \left|\frac{d\Phi}{dt}\right| = \left|\frac{dB}{dt}\right| A \approx 7.3 \times 10^{-11} \text{ V}$$

Such a small induced voltage will be completely lost within the normal electrical noise within your phone circuits.

4. (40 points) An important discovery in the 1980s was the W^+ and W^-, which are among the particles responsible for the weak nuclear force. W^+ and W^- are antiparticles of each other, and they each have a rest mass of 80.4 GeV/c^2. Say you want to create a W^+ and W^- pair by a head-on collision of an electron and a positron (e^- and e^+) into each other at speeds close to the speed of light. The rest mass of an electron (and a positron, its antiparticle) is 0.511 MeV/c^2. (Remember: 1 GeV = 1000 MeV.)

(a) As observed from the lab frame of reference, the e^- and e^+ head toward each other with equal and opposite velocities in the collision. What is the minimum time dilation factor γ that the e^- and e^+ must have in order to produce enough energy to create a W^+ and W^- pair at rest?

Answer: Energy conservation means that

$$2m_W c^2 = 2\gamma m_e c^2 \qquad \Rightarrow \qquad \gamma = \frac{m_W}{m_e} = 1.57 \times 10^5$$

This is an enormous γ; the speed of the electron must be almost c.

(b) Say that in the lab frame of reference, the electron traveled 30.0 km at a constant speed corresponding to the γ you calculated in (a). How far did it travel in its own frame of reference?

Answer: Length contraction: $\Delta L = \Delta L_0/\gamma$, therefore

$$L = \frac{3 \times 10^4 \text{ m}}{1.57 \times 10^5} = 0.191 \text{ m}$$

(c) Calculate how long it took for the electron to travel 30.0 km in the lab frame of reference. Then calculate how long this time interval was in the electrons own frame of reference.

Answer: The speed is almost c. Therefore, to travel 30 km,

$$\Delta t = \frac{\Delta L_0}{c} = \frac{3 \times 10^4 \text{ m}}{3 \times 10^8 \text{ m/s}} = 10^{-4} \text{ s}$$

For the electron,

$$\Delta t_0 = \frac{\Delta_t}{\gamma} = \frac{\Delta L}{c} = 6.37 \times 10^{-10} \text{ s}$$