Solutions to Exam 2; Phys 186

1. (50 points) Say you have a rocket that, without fuel, has a mass of $m_s = 1.0 \times 10^5$ kg. Starting from rest, the rocket needs to accelerate up to $\frac{1}{2}c$. But we also need to account for the fuel that the rocket needs to carry. As a rough estimate, say that the total mass that we have to accelerate to $\frac{1}{2}c$ is $m_s + \frac{1}{2}m_f$, where m_f is the fuel mass.

Finally, say we use the most efficient fuel possible: m_f consists of equal amounts of antimatter and matter, which annihilate. Let's (very unrealistically) assume that all of the energy we get from this annihilation goes into increasing the kinetic energy of the rocket.

(a) What is m_f?

Answer: The kinetic energy required must be supplied by the annihilation of matter and antimatter. So set these two energies equal to one another:

$$K = (\gamma - 1)\left(m_s + \frac{1}{2}m_f\right)c^2 = m_f c^2$$

Solving for m_f,

$$m_f = \frac{(\gamma - 1)m_s}{1 - \frac{1}{2}(\gamma - 1)} = 0.17 m_s = 1.7 \times 10^4 \text{ kg}$$

(b) What is the energy required to achieve $\frac{1}{2}c$?

Answer: $K = m_f c^2 = 1.5 \times 10^{21}$ J

(c) The total amount of energy produced in the United States for a year is about 10^{20} J. Given this, what does your calculations tell you about the future prospects for interstellar travel with rockets?

Answer: We need 15 times the energy the US produces in a year. It looks like rocket travel is not feasible. It requires way too much energy. The economic cost, if nothing else, is likely to be insurmountable.

(d) Before it starts accelerating, the pilot measures the length of her rocket to be 58 m. She then measures it again after achieving $v = \frac{1}{2}c$. What does she find?

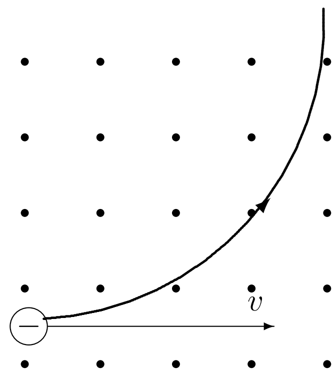
Answer: In both cases, the pilot will be measuring ΔL_0. So nothing will change: 58 m.

(e) If you had a laboratory that was isolated from outside information, you couldn't perform any experiment that could tell you that you whether were in a gravitational field or accelerating in empty space. Put that observation together with what you know about lengths and time intervals in the frame of reference of a photon traveling at the speed of light, and explain why physicists say that "spacetime is curved" and that "gravity is due to the curvature of spacetime."

Answer: In an accelerating reference frame, the path of a light beam will be curved, as with any projectile that accelerates downwards. But for light, $\gamma = \infty$. $\Delta L = \Delta L_0/\gamma = 0$ in the direction of travel, and $\Delta t_0 = \Delta_t/\gamma = 0$. Therefore light always takes the shortest path. When the path of light is curved, that means spacetime is such that the shortest path between two points in no longer a straight line—spacetime is curved! And since the effects of gravity and acceleration are indistinguishable, gravity is due to the curvature of spacetime.

2. (40 points) Let's say you did your experiment determining the charge to mass ratio of an electron, e/m_e, using only a magnetic field. You accelerate your beam of electrons, starting from rest, through an accelerating voltage V_a, and shoot them into a region with a uniform magnetic field perpendicular to their velocity. You then measure r, the radius of the arc into which the beam is bent.

(a) In the picture below, indicate the direction of the magnetic field. To the right of the picture, briefly explain your choice. The electrons come from the left; the picture shows one electron just entering the region with the magnetic field. The arc is the beam trace you see on your phosphorescent screen.



Answer:

The magnetic force must be directed upwards, toward the center of the circle. Using the right hand rule, this would mean a magnetic field into the page. But an electron has a *negative* charge, so we need to reverse the direction for a magnetic field coming out of the page.

(b) Find an equation for e/m_e in terms of what you can measure or set in the lab: V_a, r (the radius of the arc), and B (the magnitude of the magnetic field). *Hint:* You may want to remind yourself about uniform circular motion; section 6.2.

Answer: The only significant force on the electron is the magnetic force. (Gravity is too weak.) Therefore the magnetic force supplies the centripetal force for moving in a circle:

$$evB = \frac{mv^2}{r}$$

We need the speed v, which we can get from energy conservation:

$$\frac{1}{2}mv^2 = eV_a \qquad \Rightarrow \qquad v = \sqrt{\frac{2eV_a}{m}}$$

Putting these two together,

$$eB = \frac{m}{r}\sqrt{\frac{2eV_a}{m}} \qquad \Rightarrow \qquad \frac{e}{m} = \frac{2V_a}{B^2r^2}$$

3. (40 points) You have an appliance that draws a large current I when in operation. This current creates a magnetic field in the next room, which we measure to be roughly uniform with magnitude $B = kI$, with a constant $k = 0.027$ T/A. In the next room, you have another appliance, which we shall model as a simple circuit with resistance 14.0 Ω, which presents an area of $A_\perp = 0.13$ m^2 perpendicular to the magnetic field, and an area of $A_\parallel = 0.049$ m^2 parallel to the magnetic field.

Starting from $I = 0$, you switch your appliance on. The current rises at a constant rate for a time interval of 0.21 s, reaching a value of 18.0 A. After this interval, the current remains constant at 18.0 A.

Hint for the following: You might need to calculate the rate of change of the magnetic field. To do so, notice that k is a constant, so that $\frac{d}{dt}B = \frac{d}{dt}(kI) = k\frac{d}{dt}I$.

(a) Find the current induced in the circuit in the other room *just before the current is switched on.*

Answer: The current, and therefore the magnetic flux, is not changing before the appliance is switched on. Therefore no voltage is induced, and $I_{\text{induced}} = 0$.

(b) Find the current induced in the circuit in the other room *during the 0.21 seconds in which the current is rising.*

Answer: Since the current rises to 18 A from 0 in 0.21 s, the rate of change of current is $\frac{d}{dt}I = (18 - 0)/0.21 = 85.7$ A/s. The induced voltage is

$$V = \left|\frac{d\Phi}{dt}\right| = A_\perp \frac{dB}{dt} = A_\perp k \frac{dI}{dt} = 0.30 \text{ V}$$

This induces a current of

$$I_{\text{induced}} = \frac{V}{R} = 0.021 \text{ A}$$

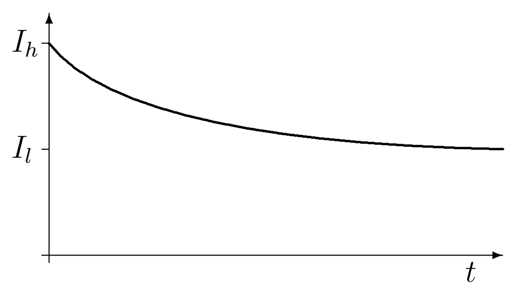
(c) Find the current induced in the circuit in the other room *after the current reaches its constant* 18.0 A *value.*

Answer: Once again, since the current is not changing, Φ is not changing, so no current is induced.

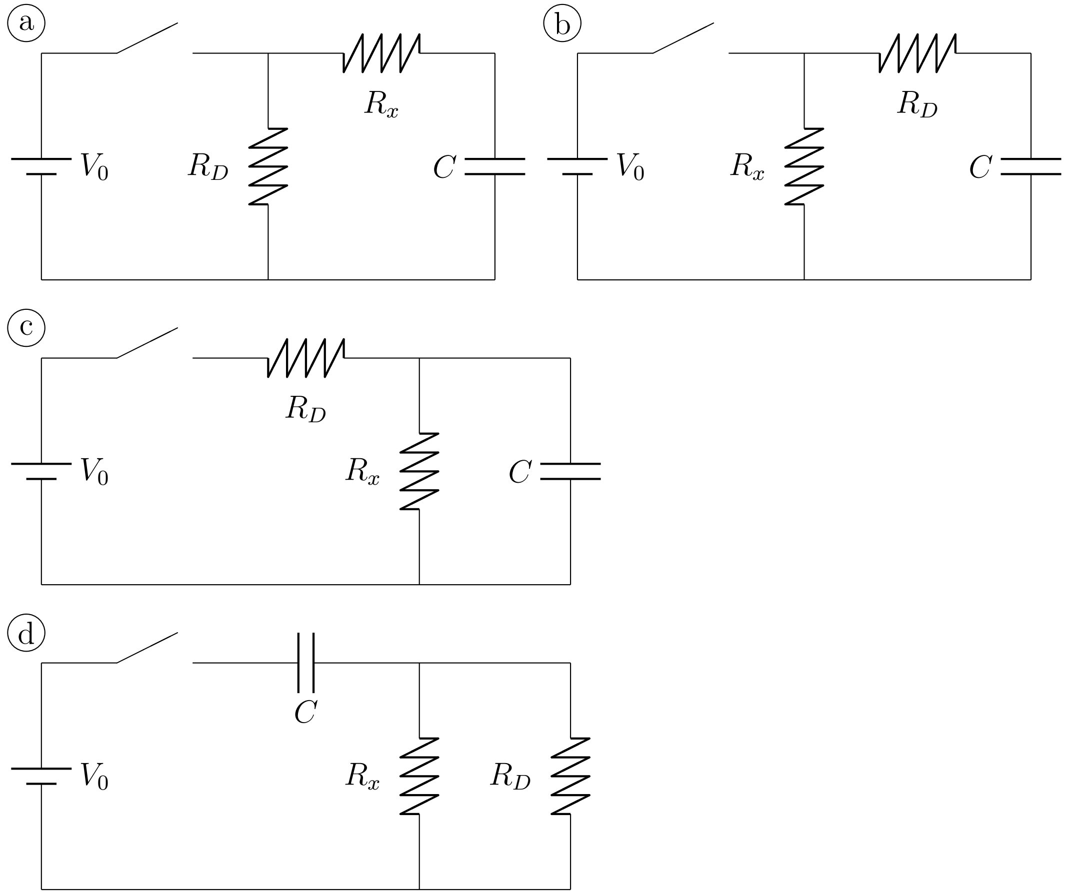
(d) If your appliance worked on an AC rather than DC power supply, would you expect it to disrupt surrounding circuits differently before, during, and after you turn the power on? Explain.

Answer: While the appliance is on, an AC appliance will be creating a constantly changing Φ, since the current it draws varies sinusoidally, and quite rapidly. Therefore an AC appliance disrupts nearby circuits continually while it is in operation, not just in the process of turning on.

4. (70 points) You've invented an electronic device that requires a quick burst of high current, I_h, when you turn it on, in order to heat it up. But you want the current to then quickly drop to a lower value, I_l, for the normal operation of the device. In other words, you want an I vs t graph that looks like the following, with $I_h > I_l > 0$.



You decide to search the Internet for a circuit that would achieve this. Unfortunately, you find four different circuits proposed to do this job for you. In these circuits, your device appears as a resistance R_D. You also need a DC battery with voltage V_0, an extra resistance R_x, and a capacitor C, whose values depend on how fast you want the initial current through R_D, I_h, to drop to I_l.



To determine which circuit will do the job, analyze all four to find I_D, the current through your device R_D. First look at their behavior at $t = 0$, immediately after the switch is closed, and there has been no time from charge to build up on the capacitor. Then look at their behavior at long times $t \gg RC$, when the capacitor has been fully charged. And then, sketch I_D vs t for all four circuits.

If you insist on numbers, use $R_D = 24\,\Omega$, $R_x = 4.2\,\Omega$, $C = 2.4\,\mu\text{F}$, $V_0 = 18.0\,\text{V}$. I'll give you a bonus **+10 points** if you do everything symbolically, using no numbers.

Answer: Circuit **a** has one junction equation, $I_0 = I_D + I_x$, and two loops, $V_0 = R_D I_D$ and $R_D I_D = R_x I_x + V_C$.

Circuit **b** has one junction equation, $I_0 = I_D + I_x$, and two loops, $V_0 = R_x I_x$ and $R_x I_x = R_D I_D + V_C$.

Circuit **c** has one junction equation, $I_D = I_C + I_x$, and two loops, $V_0 = R_D I_D + R_x I_x$ and $R_x I_x = V_C$.

Circuit **d** has one junction equation, $I_C = I_D + I_x$, and two loops, $V_0 = V_C + R_x I_x$ and $R_x I_x = R_D I_D$.

At $t = 0$, the capacitor will have $Q = 0$. Since $Q = CV$, that means that immediately after the switch is closed, they voltage across the capacitor is $V_C = 0$. The current through C, however, will not be zero, since charges are on the move to charge the capacitor up.

Using the loop and junction equations, I_D at $t = 0$ is the same for all four:

Circuit a: $I_D = V_0/R_D$. (Or $I_D = 18/24 = 0.75\,\text{A}$.)

Circuit b: $I_D = V_0/R_D$. (Or $I_D = 18/24 = 0.75\,\text{A}$.)

Circuit c: $I_D = V_0/R_D$. (Or $I_D = 18/24 = 0.75\,\text{A}$.)

Circuit d: $I_D = V_0/R_D$. (Or $I_D = 18/24 = 0.75\,\text{A}$.)

But at large t, the capacitor will have fully charged up. With no more charges moving to or from it, the current $I_C = 0$. The voltage V_C, however, will not be zero.

Using the same loop and junction equations, I_D at $t \gg RC$ is no longer the same the same for all four:

Circuit a: $I_D = V_0/R_D$. (Or $I_D = 18/24 = 0.75\,\text{A}$.)

Circuit b: $I_D = I_C = 0$.

Circuit c: $I_D = V_0/(R_D + R_x)$. (Or $I_D = 18/(24 + 4.2) = 0.64\,\text{A}$.)

Circuit d: $I_D = I_C = 0$.

So circuit **a** won't do the job, since I_D never changes.

Circuits **b** and **d** won't work, because I_D drops to 0 at long times, rather than leveling off at an I_l such that $I_h > I_l > 0$.

Circuit **c** is what you want. You start at $I_D = V_0/R_D$ and end up at $I_D = V_0/(R_D + R_x)$. And

$$\frac{V_0}{R_D} > \frac{V_0}{R_D + R_x} > 0$$