Solutions to Exam 3; Phys 186

1. (30 points) We characterize waves by their frequency, wavelength, and amplitude. Audible sound has a frequency range of 20 Hz to 20 kHz, wavelengths between 1.7 cm and 17 m, and a minimum intensity of 10^{-12} W/m^2. *Ultrasound* can be used for medical imaging, where it can resolve structures with sizes considerably less than 1 cm. The "ultra" in ultrasound must therefore refer to higher than audible frequency, wavelength, or amplitude—which one? Explain. For the range of values in question, you can take the speed of sound to be constant.

Answer: Since medical imaging should see detail with sizes less than 1 cm, we need sound with wavelengths *less* than audible wavelengths which start at 1.7 cm. Since $v = \lambda f$, and v is constant, that means that wavelength and frequency are inversely related. So smaller than audible wavelengths corresponds to *higher than audible frequency.* Amplitude is irrelevant.

2. (30 points) Here is an annoying quotation from a "quantum healing" web site:

> . . . to live healthier and longer . . . you can use the wave-particle duality from quantum physics. First, you use waves produced by your neuron vibrations. When you have an experience, e.g. saying a prayer for your loved one who is ill, relevant neurons in your brain vibrate in unison at similar frequency. These vibrations unify your deepest desire, emotion, intelligence and spiritual strengths that you can use for healing. Even though you are physically separated from your loved one, visualize that quantum fields can bring your vibrations into relationship between both of you.
>
> Second, you use boson particles. When two systems interact, they exchange bosons. We can say that bosons are particles of relationship. Bosons can merge and become one entity. When you and your partner desire for healing, visualize that both of your bosons merge to achieve the synergistic healing.

(a) Briefly explain wave-particle duality, and point out what is wrong with the quotation.

Answer: The quotation is complete drivel, with no conception of either the biology of the brain or physics. It includes the phrases "wave-particle duality" and "quantum fields" but actually does not make use of either concept.

Wave-particle duality refers to how, in quantum mechanics, all information about particles is expressed by a wave function which is a solution to a wave equation. We do not observe the wave function—we observe particles—but the behavior of the particles is random. The probability distributions for observable properties of the particles is derived from the wave function.

(b) Briefly explain the quantum picture of interactions, and point out what is wrong with the quotation.

Answer: The quotation is correct that in quantum mechanics, interactions are due to an exchange of particles. Each fundamental force has its own set of distinct exchange particles.

Talking about personal exchange particles and implying that personal relationships are just like fundamental interactions, however, is ridiculous. You cannot make a direct analogy between what is happening between subatomic particles, described by quantum physics, with interactions between large, high temperature, extremely complex entities such as humans.

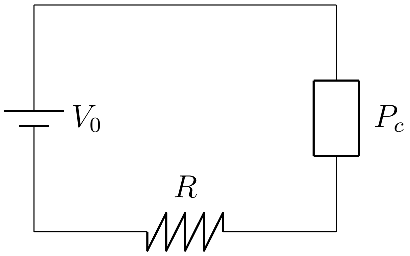
3. (30 points) There have been occasional concerns that people living close to electric power lines might have high risks of cancer due to their proximity to electromagnetic radiation emitted by the power lines. Determine, using a simple calculation, whether this concern is warranted. I will supply any physical data that you need—just ask me. I'm interested in whether you know to ask the right questions about the numbers that you might need.

Answer: Power lines carry alternating current, with a frequency of 60 Hz. That means accelerating electric charges with a 60 Hz frequency, producing predominantly 60 Hz electromagnetic waves. Photons with this frequency have an energy of

$$E = hf = 4.0 \times 10^{-32}\ \mathrm{J} = 2.5 \times 10^{-13}\ \mathrm{eV}$$

Chemical bond energies are of the order of eV's. So the photons produced by power lines have energies that are extremely smaller than bond energies. Therefore they cannot disrupt biologically important molecules such as DNA, thereby causing cancer.

4. (30 points) Electric power transmission lines are set up to minimize resistive power losses over long distances. Here is a simplified model of a circuit with a power plant, power lines, and a city consuming power. The power plant is a battery, which supplies a voltage V_0 and puts out a current I_0. The power lines are a fixed resistance R. And we will represent the city as a device that simply consumes a constant power, P_c.



(a) The power supplied by the battery is $P_0 = V_0 I_0$. Show that this is so, using the relationship of voltage to charge q and energy difference ΔU_E, and the relationship of current to charge q and time Δt.

Answer: Power is the rate that energy is extracted from the battery, $P = \frac{d}{dt}U_E$, or in our non-calculus terms, $P = \Delta U_E/\Delta t$. The relationship between voltage and energy is $qV_0 = \Delta U_E$ where q is electric charge. Current is the rate at which charges go by, so $I_0 = q/\Delta t$. Therefore

$$V_0 I_0 = \left(\frac{\Delta U_E}{q}\right)\left(\frac{q}{\Delta t}\right) = \frac{\Delta U_E}{\Delta t} = P_0$$

(b) In the circuit above, the power supplied by the battery is $P_0 > P_c$. Show that the power lost to dissipation by R becomes smaller as V_0 becomes larger. Hence power lines operate at very high voltages to minimize the loss.

Answer: Energy conservation means that the power dissipated by R and used by the city must be supplied by the battery: $P_0 = P_R + P_c$. It's a circuit with one loop and no junctions, so the same current I_0 goes through r and the battery. Therefore

$$V_0 I_0 = I_0^2 R + P_c$$

To minimize P_R, I_0 has to be as small as possible. But P_c is a constant, and the battery must supply $P_0 > P_c$. This means that lowering I_0 can only happen if V_0 is increased at the same time. So we end up with high voltage power lines.

5. (30 points) Students doing a radiation lab do a long-time measurement of the average background radiation, and find 1.2 counts/second. They then place a sample of an α-emitting isotope by their detector for a long time, and record an average of 24.2 counts/second. Exactly a year later, another group of physics students perform the same lab. They again find 1.2 counts/second as the background. And when they use the exact same α-emitting sample in the same way, their detector gives 8.7 counts/second as the activity. What is the half-life of this isotope?

Answer: To get the activities due to the isotope alone, we have to subtract the background. So $A_0 = 23$ counts/s and $A_1 = 7.5$ counts/s. The activity of a sample is proportional to how many nuclei of the radioactive isotope it contains, $A \propto N$. Therefore

$$\frac{A_1}{A_0} = \frac{N_1}{N_0} = 2^{-t/t_{1/2}}$$

Giving

$$t_{1/2} = \frac{1}{\log_2 \frac{23}{7.5}} = 0.62 \text{ years}$$

6. (50 points) The diameter of a proton is about 10^{-15} m. But the proton is not an elementary particle: it is three quarks bound by the strong nuclear force. The strong nuclear force is such that quarks very close together behave as if they are free—the force becomes negligible. But if the quarks move further apart than about the size of the proton, the strong force becomes huge. In other words, quarks are in a situation similar to being confined to a box (like a quantum dot). The mass of each quark is about 2 MeV/c^2, which is much smaller than the mass of a proton, 938 MeV/c^2. The large difference must mean that most of the mass of a proton is due to the energies of the bound quarks and $E = mc^2$! To see this, let's estimate the mass of the proton.

(a) Estimate p, the magnitude of the momentum of one of the quarks that make up a proton. *Note:* If you have a probability distribution $\mathcal{P}(z)$ for an arbitrary variable z, with standard deviation Δz, a good estimate for the average value for $|z|$ is Δz.

Answer: We can use the uncertainty principle, with $\Delta x \approx d$, the diameter of the proton. In that case, taking $\Delta x \Delta p \approx \frac{\hbar}{2}$,

$$\Delta p \approx \frac{\hbar}{2\Delta x} \approx \frac{\hbar}{2d}$$

We need an estimate for p, not Δp. But the note about distributions means that these will be close. Therefore

$$p \approx \frac{\hbar}{2d} = 5.03 \times 10^{-20}\ \text{kg} \cdot \text{m/s}$$

(b) Using $p = mv$ and your estimated p, calculate the speed v of the quark. Look carefully at this value. Does this mean that you should use the nonrelativistic expressions for the kinetic energy and momentum, $K = p^2/2m$ and $p = mv$, or the relativistic expressions $K = (\gamma - 1)mc^2$ and $p = \gamma mv \approx \gamma mc$ with $v \approx c$?

Answer: The quark mass is 2 MeV/c^2—converting the units, this is 3.56×10^{-30} kg. Therefore

$$v = \frac{p}{m} = 1.41 \times 10^{10}\ \text{m/s} > c$$

This is greater than the speed of light, which cannot happen. So we need the relativistic expressions for p and K; indeed, we can take $v \approx c$.

(c) Using your chosen equations for p and K, estimate the energy of a quark in the proton.

Answer: With $p \approx \gamma mc$, can solve for the time dilation factor γ:

$$\gamma = \frac{p}{mc} = 47.1$$

which is consistent with $v \approx c$. The energy (we might as well use the total energy now) is therefore

$$E = \gamma mc^2 = 94.2 \text{ MeV}$$

(d) Finally: there are three quarks in a proton. Estimate the total energy, and therefore the mass, of the proton. Compare your result to 938 MeV/c^2—you won't be exact, but your estimate should have the right order of magnitude (be within a factor of 10).

Answer: Multiplying 94.2 by 3, we have 283 MeV/c^2. This isn't great, but for a quick-and-dirty estimate, it actually is not at all bad. It certainly has the right order of magnitude.