Solutions to Exam 3; Phys 186

1. (40 points) You have a spaceship traveling directly away from Earth, with a relative velocity v. Ground control communicates with the spaceship by radio. At some point in the communication, ground control sends out a radio wave with wavelength λ_s and period T_s (s for "source") as measured in the Earth's frame of reference.

(a) Since the ship is moving away, when ground control observes T', the time interval between wave crests reaching the ship, they will not measure $T' = T_s$—that would be true only for a ship that was not moving. Which of the following is the correct expression for T'?

Hint 1: This is an ordinary wave question. It has nothing to do with relativity, since all time intervals in the question are those measured in a single frame of reference, that of ground control. If the question was about water waves going toward a boat moving away from the source of the waves, the answer would be the same.

Hint 2: Even if you don't know what you're doing, only one of the following answers makes any sense whatsoever. Ask yourself: When $v = 0$, what should T' be?

(a) $T' = \sqrt{vT_s}$
(b) $T' = \frac{T_s}{1-v/c}$
(c) $T' = \frac{v}{c}T_s$
(d) $T' = (c - v)T_s$
(e) $T' = \sqrt{1 - v^2T_s}$

Now explain why the answer you chose is physically correct. (Don't just tell me that all other options are obviously wrong, though this is in fact the case.)

Answer: $\frac{T_s}{1-v/c}$ is the only option that gives $T' = T_s$ when $v = 0$. Physically, what is happening is that as v increases, the wave crests are taking more time to reach the ship. When $v \geq c$, they never catch up. This equation just reproduces that behavior.

(b) Now let's bring in special relativity. T' is the period of waves reaching the ship from the point of view of an observer on Earth. Given time dilation, what is T_o, the period of the waves in the frame of reference of the observer on the spaceship? *Hint:* Be careful. Which is the frame of reference for which the events of the wave crests reaching the ship happen at the same location? The s frame (Earth) or the o frame (the ship)?

Using your result for T_o, find an equation that relates frequencies rather than periods: find an equation expressing f_o, the frequency observed on the spaceship, in terms of f_s, the source frequency, v, the relative speed, and c, the speed of light.

Mathematical hint: Your end result will be simpler if you use the fact that

$$1-(v/c)^2=(1-v/c)(1+v/c)$$

Answer: The events defining the time interval are at rest in the o frame, the frame of the spaceship. Therefore $T'=\gamma T_o$, and

$$T_o=\sqrt{1-(v/c)^2}\,\frac{T_s}{1-v/c}=\sqrt{\frac{1+v/c}{1-v/c}}T_s$$

Since $f=1/T$,

$$f_o=\sqrt{\frac{1-v/c}{1+v/c}}\,f_s$$

(c) Say you're an astrophysicist looking at light from a distant galaxy. You put the light through a diffraction grating, and find a pattern of spectral lines like that Hydrogen produces on a lab on Earth. But there is a difference: all the lines are shifted to a lower frequency, such that $f_o=\frac{1}{2}f_s$. The lines are, in other words, *redshifted.* (f_o is the observed frequency of each spectral line, and f_s is the source frequency of the line, as known from Hydrogen in the lab.)

Find the ratio v/c, where v is the relative speed of the far away galaxy. Is it moving toward Earth or away from Earth?

Answer: All this means that

$$\sqrt{\frac{1+v/c}{1-v/c}} = \frac{1}{2}$$

Therefore,

$$\frac{v}{c} = \frac{1-\frac{1}{4}}{1+\frac{1}{4}} = \frac{3}{5}$$

It is moving away.

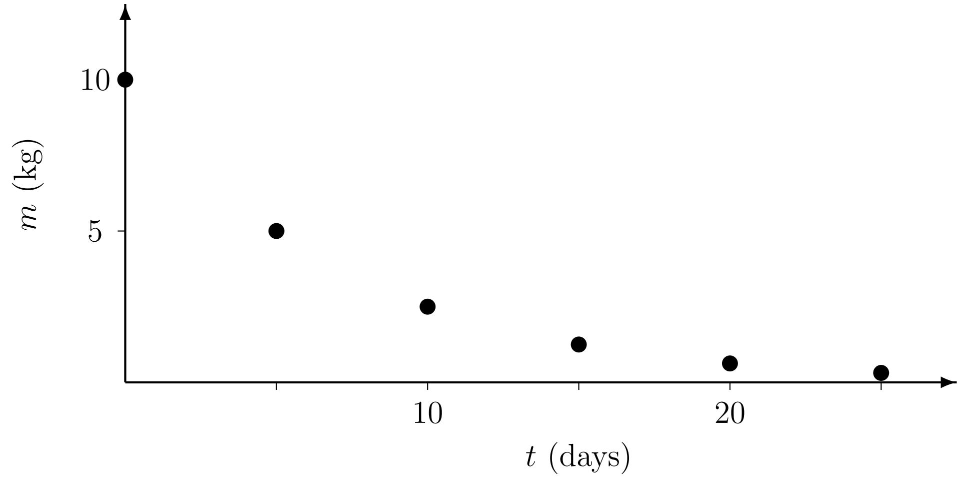
2. (40 points) Let's do half-lives.

(a) You have friend who is not a science major. She tells you that quantum mechanical events cannot be truly random. After all, randomness implies unpredictability, but physicists make precise predictions using quantum mechanics. Given the half-life, they can tell you exactly what amount of a radioactive sample will remain after a certain time. Given the energy of photons emitted from a light source, they can calculate the interference pattern observed when a diffraction grating is placed between the source and a screen. Correct your friends' misconceptions and explain what the role of randomness in quantum mechanics is.

Answer: *Individual* quantum events are random. We cannot predict when an individual nucleus will decay, or the exact path any single photon will take. But precisely *because* individual events are random, if lots and lots of individual random events take place, a very reliable statistical pattern will emerge. We get very definite half-lives and interference patterns when the number of events is about in the 10^{20}'s.

(b) You have a 10.0 kg block of radioactive material A, and at time $t = 0$, you start with all 10 kg being pure A. Sketch a graph of the amount of A that remains in the block over time. The half-life of A nuclei is 5.0 days.

Answer:



(And connect the dots.)

(c) Pick, from among the following, the correct expression for the amount of A remaining over time. Here $\tau = t_{1/2}/\ln 2$, and $m_0 = 10$ kg.

(a) $m = m_0 \cos \frac{t}{\tau}$

(b) $m = m_0 \left(1 - \frac{t}{\tau}\right)$

(c) $m = m_0 \ln \frac{t}{\tau}$

(d) $m = m_0\, e^{-\frac{t}{\tau}}$

(e) $m = \frac{m_0}{\sqrt{1-(\frac{t}{\tau})^2}}$

Answer:

(d) What is m at $t = 9.0$ days?

Answer: $m = 10\, e^{-(9 \ln 2)/5} = 2.9$ kg

3. (40 points) Recall the experiment we did in the lab, where we produced an electron beam and bent its trajectory by using electric and magnetic fields. Say we accelerated the electrons, starting from rest, by applying a voltage of 4000 V.

(a) Find v, the speed of the electrons after their acceleration is complete, by using the classical expression for kinetic energy. Then calculate the time dilation factor γ for this v, and state whether you think that this result means that you should have used the relativistic kinetic energy instead.

Answer: We need to solve for v in $\frac{1}{2}mv^2 = e(4000 \text{ V})$. We get $v = 3.75 \times 10^7$ m/s. Using this, $\gamma = 1/\sqrt{1-(v/c)^2} = 1.008 \approx 1$. Since γ is barely larger than 1, the nonrelativistic approximation should work well enough.

(b) There is a distance L between the point the electrons traveling at v emerge from the electron gun and when they hit the screen. If $L = 0.10$ m, circle what you think is a good estimate for Δx, the uncertainty in the position of the electrons along the direction in which they are traveling.

[✓ L] $\frac{hL}{4\pi}$ $\frac{h}{4\pi L}$ $\frac{c}{L}$ $\frac{\gamma c}{L}$

Now write down your estimate for the uncertainty: $\Delta x = \underline{0.10}$ m.

(c) Since the electrons are subatomic particles, we have to use quantum mechanics. Since $\Delta x < \infty$, this means that $\Delta p > 0$. In other words, we cannot assume that we know that the electrons are traveling exactly at the speed v you calculated. Estimate Δp, assuming that Δp, your uncertainty about p, is at its *minimum* possible value.

Answer: For the minimum,

$$\Delta p = \frac{h}{4\pi \Delta x} = 5.28 \times 10^{-34} \text{ kg} \cdot \text{m/s}$$

(d) Physicists usually like their electron beams to be "monochromatic"—all particles at a single wavelength. If the value of $\Delta p/p$ is small, the beam is close to monochromatic. Calculate the numerical value of p for your beam of electrons, and then find $\Delta p/p$ and decide whether this beam is almost monochromatic or not.

Answer: The momentum is $p = mv = 3.42 \times 10^{-23}$ kg·m/s. Therefore

$$\frac{\Delta p}{p} = 1.54 \times 10^{-11} \ll 1$$

Since this number is so small, the beam is nearly monochromatic.

4. (40 points) You have a circular wire, which you connect to a battery so that a current goes through the loop. You then place this circuit on a table such that the current goes around the loop in a counterclockwise direction when looked at from above. Finally, you take a small superconducting cube, and place it exactly above the center of your current loop. You find it levitates.

(a) Sketch a view of the loop from above, and also show a sideways view with the y-axis as the upward direction. Draw in roughly what the current and the magnetic field looks like. Add a small box to indicate the location in which you will place the superconducting cube.

Answer: This is the typical magnetic field due to a circular current loop, as discussed and illustrated in the textbook. With a counterclockwise current, the magnetic field directly above the loop will be upward.

(b) The currents that arise within the superconductor are such that *within the superconductor*, they create a magnetic field that is exactly equal and opposite to the external field produced by the current loop. Sketch, using appropriate viewpoints, pictures that give a qualitative idea of the direction of the currents circulating within the superconductor once you place it above the center of the current loop.

Answer: To cancel out a directly upward magnetic field, the currents within the superconductor must produce a downward magnetic field. This requires currents that circulate in a clockwise fashion when seen from above.

(c) Now sketch the magnetic forces due to the external magnetic field from the current loop on the currents within the superconductor. This should help you see why the levitation happens. Why does the cube levitate?

Answer: Wires carrying parallel currents attract. This situation, with the clockwise and counterclockwise currents, is like a case where the currents are in opposite directions, so the objects carrying the currents are repelled. If you do the usual right hand rule with the current from (b) within the upward magnetic field from (a), you will find that on each segment of current, you will get a force that is inward in the xy-plane, but has an upward component as well. The forces in the xy-plane cancel out, leaving a net upward, repulsive force. This force acts against gravity, when it is equal in magnitude to the cube's weight, the cube will be suspended in mid-air.

(d) Will the cube levitate only at a single height above the loop, or will it levitate at any height you happen to place it? Explain.

Answer: The repulsive magnetic force in (c) depends on the magnitude of the magnetic field, which changes with the cube's height. Therefore the magnetic repulsion will not be able to cancel out the weight at any height you happen to place it.

5. (40 points) A neutral pion, π^0, is a bound state of an up quark, u, and an anti-up quark $\bar{u}$. The u and $\bar{u}$ are antiparticles of each other: they have the same mass and spin, but opposite charges. The π^0 has a mass of 135 MeV/c^2, and an electron has a mass of 0.511 MeV/c^2.

(a) We want to create a π^0 by colliding an electron and a positron at high speeds, so that $e^- + e^+ \to \pi^0$. (A positron and electron are antiparticles of each other.) This is possible *if* all conservation laws are obeyed. You have learned about the conservation of linear momentum, angular momentum, energy, and charge in this course.

Angular momentum is the hardest for you to check, so I'll do it for you: π^0 has spin 0, therefore if the electron that meets the positron have opposite spins, angular momentum will conserved. So angular momentum conservation does not prevent $e^- + e^+ \to \pi^0$.

Check if linear momentum, energy, and charge conservation prevent $e^- + e^+ \to \pi^0$ or not.

Answer: Linear momentum conservation is easily satisfied, for example if the e^- and e^+ have velocities that are equal in magnitude but opposite in direction. The resulting pion would then have zero velocity, conserving momentum.

Energy will be conserved, as long as the e^- and e^+ have large enough speeds so that $2\gamma m_e c^2 = m_\pi c^2$.

Since e^- and e^+ are antiparticles, their total charge is zero. And the charge of a neutral π^0 is also zero. So charge is conserved.

Therefore $e^- + e^+ \to \pi^0$ is possible.

(b) Say you try to isolate a u quark by removing it from the $\bar{u}$ in a π^0, just like you could obtain an isolated electron by providing enough energy to ionize a Hydrogen atom. You find that this does not happen. Instead, if you add an amount of energy E to a pion, you might get a result like

$$\pi^0 + E \to \pi^0 + \pi^0 \qquad \text{or} \qquad \pi^0 + E \to \pi^0 + e^- + e^+$$

So you might end up with multiple pions and other particles, but no matter how large E is, you will not isolate a quark. Explain why this is so, using the equivalence of mass and energy ($E = mc^2$) and the fact that the strong force between quarks grows larger as the distance between quarks increases.

Answer: Trying to separate a quark and an antiquark means pumping indefinitely large amounts of energy into the π^0, since the strong force increases with distance. Very soon, E will be large enough to allow creation of particle-antiparticle pairs, so you will see a bunch of particle-antiparticle pairs added to the π^0, rather than an isolated quark.