Homework Solutions 3 (Schroeder Chapter 3)

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(a) Melting (at constant temperature) increases entropy by

$$\Delta S_a = \frac{Q}{T_i} = \frac{mL}{T_i} = 36.6 \text{ J/K}$$

(b) Integrate for the entropy due the temperature increase:

$$\Delta S_b = C \int_{T_i}^{T_f} \frac{dT}{T} = C \ln \frac{T_f}{T_i} = 11.0 \text{ J/K}$$

(c) The kitchen is at a constant temperature, and the heat transferred to the ice and then water is lost by the kitchen. So the kitchen entropy changes by

$$\Delta S_c = -\frac{mL + mC(T_f - T_i)}{T_f} = -44.1 \text{ J/K}$$

(d) Add all these to get the change in entropy for the universe:

$$\Delta S_{\text{total}} = \Delta S_a + \Delta S_b + \Delta S_c = 3.5 \text{ J/K}$$

This is positive, as it should be.

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(a) Using Ω,

$$S = k \ln \Omega = k \left(q \ln \frac{q+N}{q} + N \ln \frac{q+N}{N} \right)$$

(b) Taking the derivative,

$$\begin{aligned} \frac{1}{T} &= \frac{\partial S}{\partial U} = \frac{1}{\epsilon}\frac{\partial S}{\partial q} = \frac{k}{\epsilon}\frac{\partial}{\partial q}\left[(q+N)\ln(q+N) - q\ln q - N\ln N\right] \\ &= \frac{k}{\epsilon}\left[\ln(q+N) + \frac{q}{q+N} - \ln q - \frac{q}{q} + \frac{N}{q+N}\right] = \frac{k}{\epsilon}\ln\left(1 + \frac{N}{q}\right) \end{aligned}$$

Therefore

$$T = \frac{\epsilon}{k \ln(1 + N\epsilon/U)}$$

(c) Inverting,

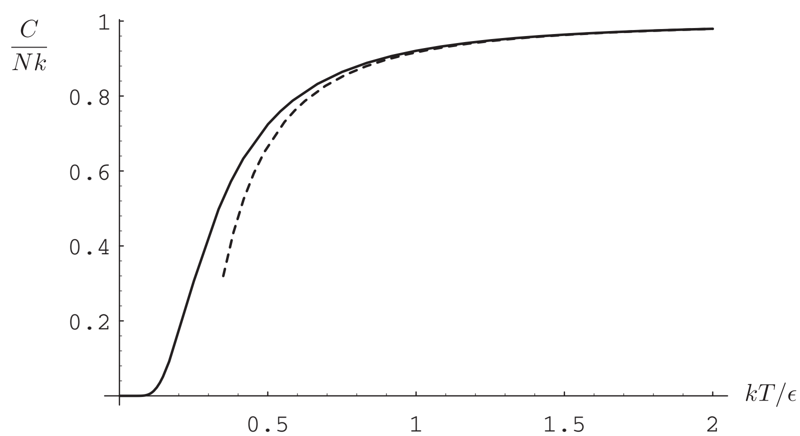
$$U = \frac{N\epsilon}{e^{\epsilon/kT} - 1}$$

$$C = \frac{\partial U}{\partial T} = \frac{N\epsilon^2}{kT^2}\frac{e^{\epsilon/kT}}{(e^{\epsilon/kT} - 1)^2}$$

(d) When $T \to \infty$, $kT \gg \epsilon$ and $e^{\epsilon/kT} \approx 1 + \epsilon/kT$. So we get

$$C \approx \frac{N\epsilon^2}{kT^2}\frac{1}{(\epsilon/kT)^2} = Nk$$

(e) The solid line is the exact result, the dashed line is the approximation in (f).



(f) With $x = \epsilon/kT$,

$$\begin{aligned}\frac{C}{Nk} &\approx \frac{x^2(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3)}{(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 - 1)^2} \\ &\approx \frac{1 + x + \frac{1}{2}x^2}{1 + x + \frac{7}{12}x^2} \approx 1 - \frac{1}{12}x^2\end{aligned}$$

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(a) Since each link is oriented toward the right or the left, this is just like a chain of spins that are up or down. We've done that;

$$S = k \ln \binom{N}{N_R} \approx N \ln N - N_R \ln N_R - (N - N_R) \ln(N - N_R)$$

(b) $L = l(N_R - N_L) = l(2N_R - N)$.

(c) $dU = T\,dS + F\,dL$.

(d) The identity gives

$$F = -T\left(\frac{\partial S}{\partial L}\right)_U \quad \Rightarrow \quad F = \frac{\partial S}{\partial N_R}\frac{\partial N_R}{\partial L} = \frac{\partial S}{\partial N_R}\frac{1}{2l}$$

Using the expressions for S and L,

$$F = -\frac{kT}{2l}\ln\left(\frac{N - N_R}{N_R}\right) = -\frac{kT}{2l}\ln\left(\frac{2}{L/Nl + 1} - 1\right)$$

(e) With $L \ll Nl$,

$$\frac{2}{L/Nl + 1} - 1 \approx 1 + \frac{2L}{Nl} \quad \Rightarrow \quad F \approx \frac{kT}{Nl^2}L$$

(f) $F \propto T$. At high temperatures, the links are more randomly oriented, so the chain will tend to shorten.

(g) Adiabatic means no entropy change. A decrease in the S we calculated here, because the configuration becomes less random, will have be compensated by an increase in the entropy due to the vibration: the temperature will increase.

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(a) The energy of the gas is $U = U_0 + Nmgz$, where U_0 is for an ideal gas at $z = 0$. So

$$\mu(z) = \left(\frac{\partial U}{\partial N}\right)_{S,V} = \mu_0 + mgz$$

But $\mu_0 = \partial U_0/\partial N$ is given in the textbook, equation (3.63).

(b) Equilibrium means $\mu_z = \mu_0$. Therefore

$$-kT\ln\left[\frac{V}{N(z)}\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right] + mgz = -kT\ln\left[\frac{V}{N(0)}\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right]$$

Canceling out the terms that appear in both logarithms,

$$kT\ln N(z) + mgz = kT\ln N(0) \quad \Rightarrow \quad N(z) = N(0)\,e^{-mgz/kT}$$

Additional chapter 4 problems

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(a) The work done *by* the gas on the outside world is $-W$. So the W for the heat engine is the area enclosed by the rectangle,

$$W = (P_2 - P_1)(V_2 - V_1) = 2P_1V_1$$

Steps A and B require heat going into the gas; these give Q_h. For each step, use $Q = C_V\Delta T$ or $Q = C_P\Delta T$ as appropriate.

$$Q_h = \frac{5}{2}V_1(P_2 - P_1) + \frac{7}{2}P_2(V_2 - V_1) = \frac{33}{2}P_1V_1$$

So

$$e = \frac{W}{Q_h} = \frac{4}{33} = 12\%$$

(b) The lowers temperature, T_c, is at the bottom left of the rectangle; T_h is at the upper right. You can see that since $PV = NkT$, $T_h = 6T_c$. Therefore

$$e_{\max} = 1 - \frac{1}{6} = 83\%$$

16 If you use all the work from the imagined HE to drive a Carnot R, you will find that this is equivalent to heat flowing from the cold reservoir to the hot reservoir, with no work being done. Heat cannot spontaneously flow from cold to hot.