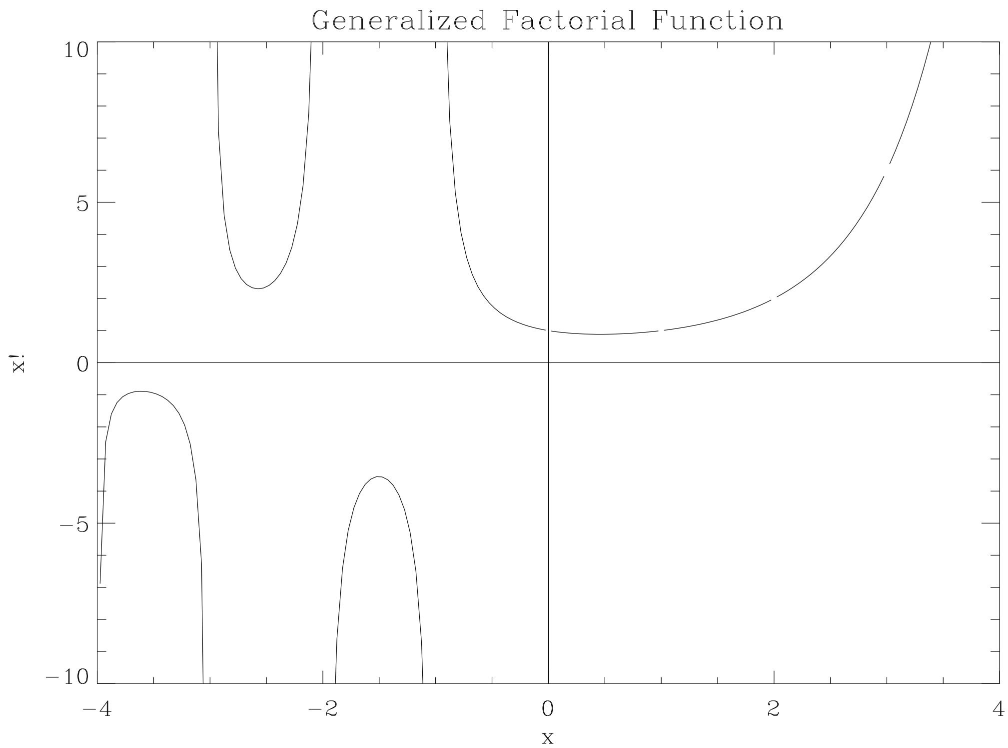
Homework Solutions, Chapter 11

Special However you found $\Gamma(x)$ or $x!$, its plot should look like:



Using a computer program such as Mathematica, you can also get:

$$\Gamma(1+i\pi) = 0.011 + 0.030\,i$$

$$\Gamma(-3.5-i) = 0.0045 - 0.026\,i$$

$$\Gamma(3i) = 0.011 - 0.0064\,i$$

5.1 $\Gamma(\frac{3}{2}) = \frac{1}{2}\Gamma(\frac{1}{2}) = \frac{\sqrt{\pi}}{2}$. $\Gamma(-\frac{1}{2}) = -2\Gamma(\frac{1}{2}) = -2\sqrt{\pi}$. $\Gamma(-\frac{3}{2}) = -\frac{2}{3}\Gamma(-\frac{1}{2}) = \frac{4\sqrt{\pi}}{3}$.

11.3 The log of Stirling's formula is:

$$\ln N! \sim N\ln N - N + \frac{1}{2}(\ln 2\pi + \ln N)$$

For $N = 10^{23}$, the first two terms are $10^{23}(23\ln 10 - 1) \approx 5.196 \times 10^{24}$. The remaining terms add up to only about 27.4, and so are completely negligible.

11.4 Replacing all factorials with Stirling's formula,

$$\lim_{n\to\infty} \frac{(2n)!\sqrt{n}}{2^{2n}(n!)^2} = \lim_{n\to\infty} \frac{(2n)^{2n}e^{-2n}\sqrt{4\pi n}\sqrt{n}}{2^{2n}n^{2n}e^{-2n}2\pi n} = \frac{1}{\sqrt{\pi}}$$