Solutions to Exam 1; Phys 186

1. (30 points) You have a mass m attached to a frictionless spring with spring constant k, and you set the mass oscillating. In the following list of variables that might describe the resulting motion, draw a circle around those that depend on the mass m:

amplitude, wavelength, <u>period</u>, phase, diffraction

Sketch a graph of this dependence (with m on the horizontal axis) for each variable you circle.

Answer: The amplitude and phase have to do with initial conditions; they do not depend on the mass. Oscillations are not waves; they have no wavelength. Diffraction also applies to waves only.

The period is $T = 2\pi\sqrt{m/k}$, therefore $T \propto \sqrt{m}$. The graph should show a square root dependence.

2. (15 points) If you get a sonic boom due to constructive interference, can you get areas where zones of destructive interference passes over the surface as well? If so, if you're within such an anti-boom, would it silence the other sources of sound present? Explain.

Answer: Yes, you can get areas of destructive interference where the intensity of the sound due to the supersonic aircraft decreases. But this would not silence other sources of sound in the area, which typically have different wavelengths and random phase relationships.

3. (15 points) Could you get destructive or constructive interference between a red laser beam and a green laser beam? Explain.

Answer: No. You need identical (or very close) wavelengths to get stable, persistent interference patterns. Red and green have very different wavelengths.

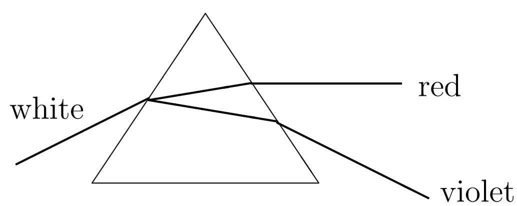
4. (15 points) Why do noise-canceling headphones fail to cancel out an annoying conversation you don't want to overhear? Explain.

Answer: Noise canceling works with steady background noises that can be eliminated by generating an inverted wave and producing destructive interference. Conversations involve non-steady, unpredictable waveforms.

5. (15 points) When underwater, you don't see the colors of objects outside the water change. How is this observation evidence that the color-sensitive cone cells in your eyes respond to frequency rather than wavelength? Explain.

Answer: The wavelengths of light waves shorten underwater, as the speed of light slows. The frequency does not change. Therefore we must be responding to frequency, not wavelength.

6. (15 points) If you see a prism separating color such that red is refracted less than violet, can you conclude that its index of refraction n increases as a function of frequency f, or does it decrease? Explain.



Answer: The larger n, the larger the refraction—the bend in the ray of light. Since violet is refracted more, and violet is the high frequency end of the visible spectrum, $n(f)$ must increase with frequency f.

7. (15 points) In some fiction, the protagonists get shrunk to the size of insects, and try to talk with unshrunken people. Assuming the shrinking process involves nothing but a scaling down (there's no speeding up or anything), would the shrunken people's speech be understood by those who remain normal? Explain.

Answer: The wavelength of sound emitted depends on the size of the source. Shrunken people would produce sounds at a smaller wavelength and thus higher frequency. If they're shrunken small enough, their high-pitched sounds can go beyond the range of human hearing.

8. (20 points) You have sound waves in air, with a frequency of f and speed of v_a, incident on a solid surface. The angle between the normal to the interface and the direction of wave propagation in air is θ_a. The speed of sound in the solid is v_s. Find an equation for θ_s, the angle the sound transmitted into the solid makes between the normal to the interface and its new direction of propagation. (*Hint:* Start by drawing a diagram of the situation, and ask me to look at it to see if it's right.)

Answer: This is exactly the same situation as with the refraction of light waves, except that the waves here are sound waves. With light, the equation describing refraction at an interface is $n_1 \sin\theta_1 = n_2 \sin\theta_2$, but the index of refraction n depends on the speed of light. Let's write this equation, then, in way that does not depend on c:

$$\frac{c}{v_1}\sin\theta_1 = \frac{c}{v_2}\sin\theta_2 \qquad \Rightarrow \qquad \frac{1}{v_1}\sin\theta_1 = \frac{1}{v_2}\sin\theta_2$$

In this version of the equation, everything depends on the wave speeds alone. That will apply to sound just as well, so

$$\frac{1}{v_a}\sin\theta_a = \frac{1}{v_s}\sin\theta_s \qquad \Rightarrow \qquad \theta_s = \sin^{-1}\left(\frac{v_s}{v_a}\sin\theta_a\right)$$

9. (40 points) You have an electric dipole arranged on the x-axis: a $+q$ charge at $x = +a$ and a $-q$ charge at $x = -a$. The charges are connected by a rigid rod, so the distance between them never changes.

(a) Calculate the electric field created by this dipole on a point on the x-axis, for $x > a$. Get both magnitude and direction.

Answer: At $x > a$, the distance to $+q$ is $x - a$ and the distance to $-q$ is $x + a$. The magnitudes of the electric field due to the two charges are

$$E_+ = \frac{kq}{(x-a)^2} \qquad\qquad E_- = \frac{kq}{(x+a)^2}$$

$E_+ > E_-$, because $+q$ is closer to points where $x > a$.

Note that $\vec{E}_+$ points in the $+x$ direction, and $\vec{E}_-$ points in the $-x$ direction. Both electric fields are along the x-axis, with no y component.

So we can add them together without worrying about trigonometry. The total electric field is

$$E = E_+ - E_- = kq\left[\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2}\right]$$

The direction of $\vec{E}$ for $x > a$ is in the $+x$ direction.

(b) Calculate the force the first dipole exerts on another dipole further down the x-axis. Get both magnitude and direction.

Answer: Use the electric field found for $x = 7a$ and $x = 9a$, and multiply by the charges at those locations:

$$F_{7a} = kq(-q)\left[\frac{1}{(6a)^2} - \frac{1}{(8a)^2}\right] = -0.012\,\frac{kq^2}{a^2}$$

$$F_{9a} = kq(+q)\left[\frac{1}{(8a)^2} - \frac{1}{(10a)^2}\right] = 0.0056\,\frac{kq^2}{a^2}$$

The total force is

$$F = -0.0064\,\frac{kq^2}{a^2}$$

The $-$ sign indicates a force in the $-x$ direction: the second dipole is attracted by the first.

(c) You now have the second dipole oriented perpendicular to the first:

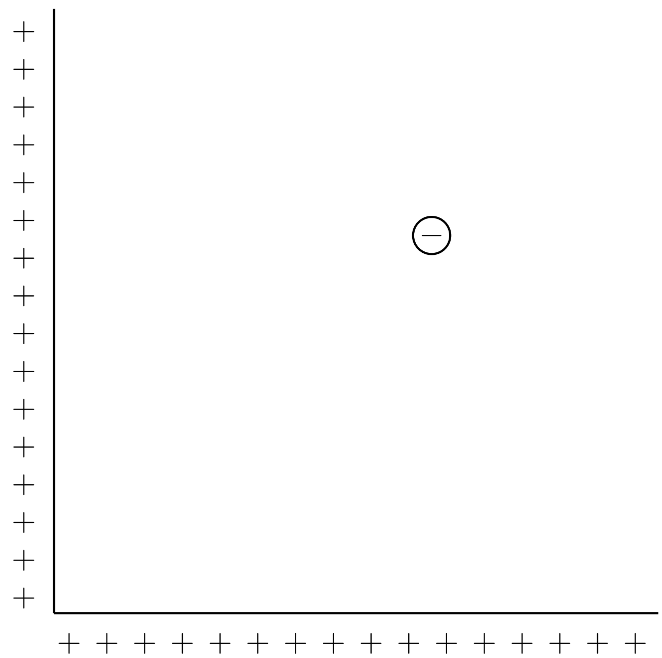
Qualitatively sketch the forces on this second dipole due to the first dipole. Also indicate in what direction (clockwise or counterclockwise) it rotates. Very briefly explain why.

Answer: The $-q$ on the second dipole will be attracted toward the first dipole, because the $+q$ on the first dipole is closer. The $+q$ on the second dipole will be repelled away from the first dipole, again because the $+q$ on the first dipole is closer. This pair of forces will produce a counterclockwise rotating torque. So the second dipole will tend to align with the first one.

(d) What can you conclude about dipole-dipole interactions from this problem? Do dipoles attract or repel one another? How do they orient themselves relative to each other?

Answer: Dipoles attract, and they orient themselves such that arrows you draw from the $-$ to the $+$ charges in each dipole are parallel to one another.

10. (20 points) Draw equipotential lines and electric field lines for the following configuration of charges:



Answer: The equipotential lines will be close to circular around the $-$ charge, nearly parallel to the plates close to the $+$ plates. In between, they will morph into one another.

The electric field lines will be perpendicular to the equipotential lines, going from $+$ to $-$. They will therefore originate from the $+$ plates, perpendicular to the plates at first, but then they will bend to connect to the $-$ charge.

None of the equipotential lines can cross another equipotential line: there's only one voltage at any point in space. None of the electric field lines can cross another field line: there is only one electric field vector at any point in space.