

**Note:** You can ask me for help; for example, have me check if an answer is correct.

**1. (50 points)** Say you have a rocket that, without fuel, has a mass of  $m_s = 1.0 \times 10^5$  kg. Starting from rest, the rocket needs to accelerate up to  $\frac{1}{2}c$ . But we also need to account for the fuel that the rocket needs to carry. As a rough estimate, say that the total mass that we have to accelerate to  $\frac{1}{2}c$  is  $m_s + \frac{1}{2}m_f$ , where  $m_f$  is the fuel mass.

Finally, say we use the most efficient fuel possible:  $m_f$  consists of equal amounts of antimatter and matter, which annihilate. Let's (very unrealistically) assume that all of the energy we get from this annihilation goes into increasing the kinetic energy of the rocket.

(a) What is  $m_f$ ?

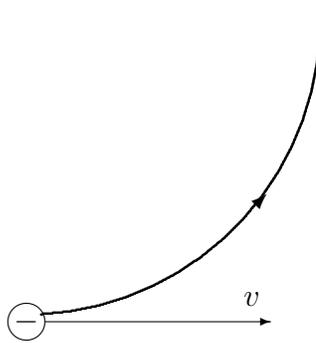
(b) What is the energy required to achieve  $\frac{1}{2}c$ ?

(c) The total amount of energy produced in the United States for a year is about  $10^{20}$  J. Given this, what does your calculations tell you about the future prospects for interstellar travel with rockets?

- (d) Before it starts accelerating, the pilot measures the length of her rocket to be 58 m. She then measures it again after achieving  $v = \frac{1}{2}c$ . What does she find?
- (e) If you had a laboratory that was isolated from outside information, you couldn't perform any experiment that could tell you that you were in a gravitational field or accelerating in empty space. Put that observation together with what you know about lengths and time intervals in the frame of reference of a photon traveling at the speed of light, and explain why physicists say that "spacetime is curved" and that "gravity is due to the curvature of spacetime."

2. (40 points) Let's say you did your experiment determining the charge to mass ratio of an electron,  $e/m_e$ , using only a magnetic field. You accelerate your beam of electrons, starting from rest, through an accelerating voltage  $V_a$ , and shoot them into a region with a uniform magnetic field perpendicular to their velocity. You then measure  $r$ , the radius of the arc into which the beam is bent.

- (a) In the picture below, indicate the direction of the magnetic field. To the right of the picture, briefly explain your choice. The electrons come from the left; the picture shows one electron just entering the region with the magnetic field. The arc is the beam trace you see on your phosphorescent screen.



- (b) Find an equation for  $e/m_e$  in terms of what you can measure or set in the lab:  $V_a$ ,  $r$  (the radius of the arc), and  $B$  (the magnitude of the magnetic field). *Hint:* You may want to remind yourself about uniform circular motion; section 6.2.

**3. (40 points)** You have an appliance that draws a large current  $I$  when in operation. This current creates a magnetic field in the next room, which we measure to be roughly uniform with magnitude  $B = kI$ , with a constant  $k = 0.027$  T/A. In the next room, you have another appliance, which we shall model as a simple circuit with resistance  $14.0\ \Omega$ , which presents an area of  $A_{\perp} = 0.13\ \text{m}^2$  perpendicular to the magnetic field, and an area of  $A_{\parallel} = 0.049\ \text{m}^2$  parallel to the magnetic field.

Starting from  $I = 0$ , you switch your appliance on. The current rises at a constant rate for a time interval of  $0.21$  s, reaching a value of  $18.0$  A. After this interval, the current remains constant at  $18.0$  A.

*Hint for the following:* You might need to calculate the rate of change of the magnetic field. To do so, notice that  $k$  is a constant, so that  $\frac{d}{dt}B = \frac{d}{dt}(kI) = k\frac{d}{dt}I$ .

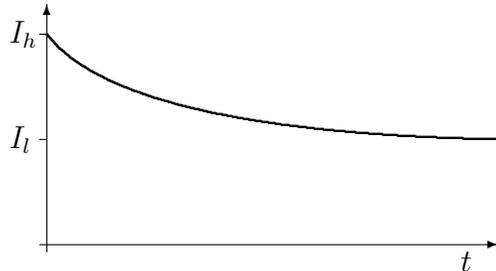
(a) Find the current induced in the circuit in the other room *just before the current is switched on*.

(b) Find the current induced in the circuit in the other room *during the 0.21 seconds in which the current is rising*.

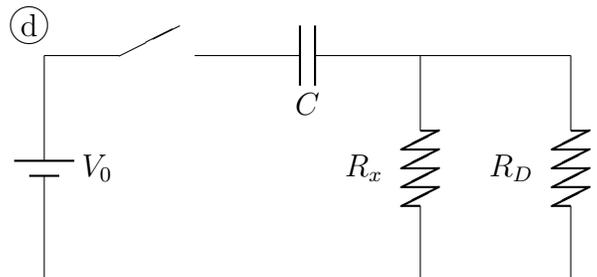
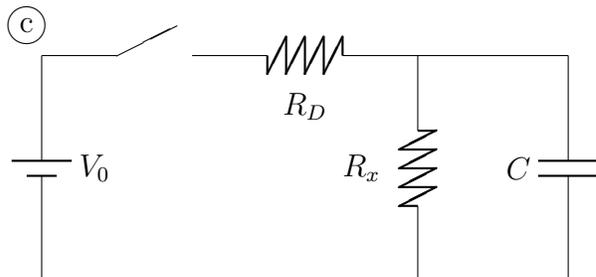
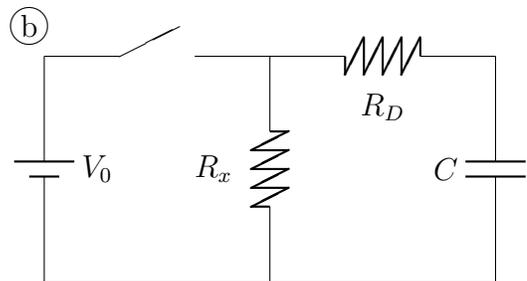
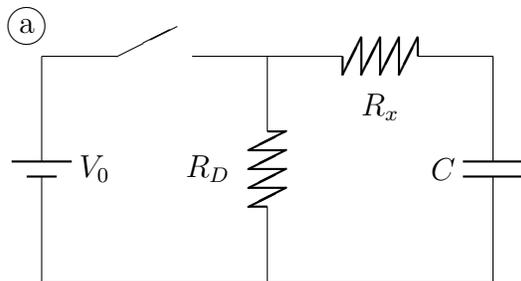
(c) Find the current induced in the circuit in the other room *after the current reaches its constant 18.0 A value.*

(d) If your appliance worked on an AC rather than DC power supply, would you expect it to disrupt surrounding circuits differently before, during, and after you turn the power on? Explain.

4. (70 points) You've invented an electronic device that requires a quick burst of high current,  $I_h$ , when you turn it on, in order to heat it up. But you want the current to then quickly drop to a lower value,  $I_l$ , for the normal operation of the device. In other words, you want an  $I$  vs  $t$  graph that looks like the following, with  $I_h > I_l > 0$ .



You decide to search the Internet for a circuit that would achieve this. Unfortunately, you find four different circuits proposed to do this job for you. In these circuits, your device appears as a resistance  $R_D$ . You also need a DC battery with voltage  $V_0$ , an extra resistance  $R_x$ , and a capacitor  $C$ , whose values depend on how fast you want the initial current through  $R_D$ ,  $I_h$ , to drop to  $I_l$ .



To determine which circuit will do the job, analyze all four to find  $I_D$ , the current through your device  $R_D$ . First look at their behavior at  $t = 0$ , immediately after the switch is closed, and there has been no time from charge to build up on the capacitor. Then look at their behavior at long times  $t \gg RC$ , when the capacitor has been fully charged. And then, sketch  $I_D$  vs  $t$  for all four circuits.

If you insist on numbers, use  $R_D = 24 \Omega$ ,  $R_x = 4.2 \Omega$ ,  $C = 2.4 \mu\text{F}$ ,  $V_0 = 18.0 \text{ V}$ . I'll give you a bonus +10 points if you do everything symbolically, using no numbers.

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