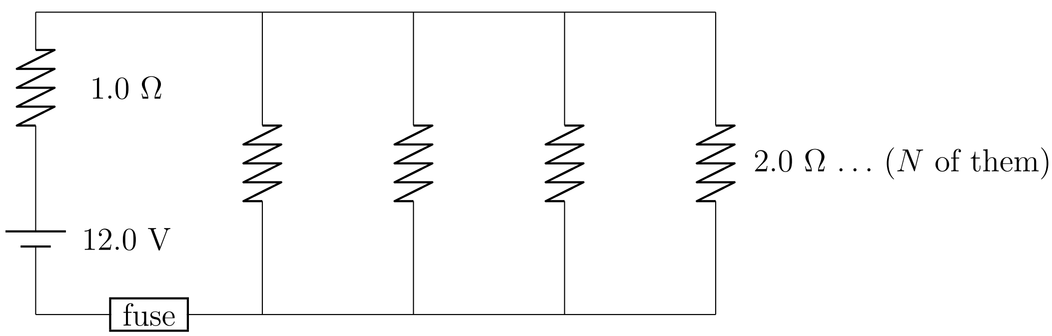
Solutions to Exam 2; Phys 186

1. (70 points) You have a circuit with a fuse in it to limit the current drawn from the battery, which supplies power to N 2.0 Ω resistors hooked in parallel. The 1.0 Ω resistor represents the internal resistance of the battery. The fuse acts like an ideal wire until a maximum current goes through it, at which point it burns up and breaks the circuit.



(a) Write down the junction, loop, and resistor equations for the circuit. Note that this is not as complicated as it looks, since many of your equations will be simplified to become identical. Ask me what I mean if you're confused.

Answer: Call the current from the battery I_0 and the currents through the resistors I_1 through I_N. There is a single junction equation, when the battery current splits into N parts:

$$I_0 = I_1 + I_2 + \ldots + I_N$$

The other junction gives the same equation, since all the split currents rejoin.

The resistor equations are $V = RI$ for each; it's easiest to directly use these in the loop equations. There are N independent loops in the circuit. The first loop equation is

$$12 \text{ V} = (1\,\Omega)I_0 + (2\,\Omega)I_1$$

The $N - 1$ other loop equations are

$$(2\,\Omega)I_1 = (2\,\Omega)I_2, \quad (2\,\Omega)I_2 = (2\,\Omega)I_2, \quad \ldots \quad (2\,\Omega)I_{N-1} = (2\,\Omega)I_N$$

All this means is that the split currents are all equal:

$$I_1 = I_2 = \ldots = I_N$$

So we end up with two equations

$$I_0 = N\, I_1, \qquad 12 \text{ A} = I_0 + 2\, I_1$$

and three unknowns, I_0, I_1, and N.

(b) If the fuse blows when the current through it is more than 10.0 A, what is the maximum number N_{max} of 2.0 Ω resistors that can be hooked up in this circuit?

Answer: We just need to solve our equations for when $I_0 = 10$ A. In that case, $I_1 = 10/N_{\text{max}}$ A, and we are left with

$$12 \text{ A} = 10 \text{ A} + 2\, \frac{10}{N_{\text{max}}} \text{ A}$$

which means that $N_{\text{max}} = 10$.

(c) Could you increase N_{max} by hooking up a capacitor in series with the 1.0 Ω resistor? In parallel? Explain.

Answer: If you hooked up a capacitor in series, it would charge up, and once it was fully charged, no more charges would be moving on that wire: the current I_0 would become zero. That would not help.

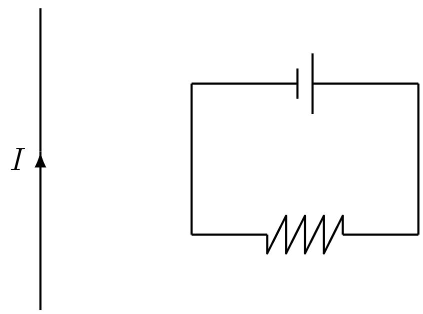
In parallel, again, the capacitor would charge up, and no current would go through afterwards. But that would just leave us with an equivalent of the original circuit after a while, so N_{max} would not be affected.

In other words, a capacitor would not do the job, however it is connected.

(d) What does this problem tell you about the consequences of hooking up lots of appliances to a single wall socket by using extension cords?

Answer: If you keep adding appliances, your fuse or circuit breaker will have to break the circuit so you don't draw too much current and start a fire.

2. (50 points) You have a long current-carrying wire, and a circuit next to it:



(a) In the picture, draw in the magnetic field produced by the long wire with current I, and indicate the current direction in the circuit.

Answer: By the right hand rule, the magnetic field produced by the long wire will be into the page where the circuit is located. The field strength will decrease as you go father from the wire, which you can indicated by drawing $\times$ symbols less densely as you move away.

The current in the circuit will be clockwise, since the + end of the battery is on the right.

(b) Will the circuit be attracted to the long wire, repelled by it, or will it feel no force? State your reasoning.

Answer: By the right hand rule, the magnetic force on the current in the part of the circuit closest to the wire will be toward the left. The segment with the battery will feel a force upward. The far segment will feel a rightward force. And the segment with a resistor will feel a downward force.

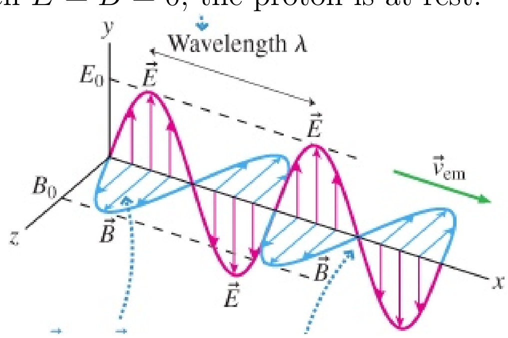
Observing the symmetry of the situation, the upward and downward forces will be equal and opposite: they will cancel. The magnitude of the leftward force, however, will be larger than that of the rightward force. This is because while the currents are the same, the magnetic field magnitude is smaller on the side farther away from the long wire.

So when you add all the forces up, you will end up with a total force toward the left, which means that the circuit will be attracted to the wire.

(c) If you let the circuit move according to the total magnetic force that might be acting on it, will any extra voltage V_{extra} be induced in the circuit? State your reasoning.

Answer: Yes. If the circuit moves toward the left, its area and orientation won't change, but the magnetic field will become stronger. Therefore the magnetic flux Φ will change, and a non-zero rate of change will mean a non-zero V_{extra} induced in the circuit.

3. (40 points) You have an electromagnetic wave as shown (Figure 25.25 in your textbook). Say you have a proton placed on the x-axis, such that when $\vec{E} = \vec{B} = 0$, the proton is at rest.



(a) Along what axis will the electric force on the proton be directed? If the electric force was the only force on the proton, what would its motion be like?

Answer: The electric force $\vec{F}_E = q\vec{E}$, along the y-axis. Therefore the electric force felt by the proton will vary sinusoidally. This is exactly like the spring force on a mass connected to a spring, and therefore the proton will undergo simple harmonic motion (oscillate sinusoidally) up and down the y-axis.

(b) Assume the proton starts to move under the influence of the electric force. In that case, along what axis will the magnetic force on the proton be directed? What sort of motion would the combination of electric and magnetic forces produce?

Answer: If the proton moves up and down along the y-axis, it will have a non-zero velocity component that is $\perp$ to the magnetic field. Therefore, the magnetic force $\vec{F}_B$, which is $\vec{F}_B \perp \vec{B}$ and $\vec{F}_B \perp \vec{v}$, will be along the x-axis. Since $\vec{F}_B \perp \vec{F}_E$, the y-axis motion described in (a) won't be affected. The end result is sinusoidal oscillations along the x-axis as well as the y-axis.

4. (40 points) You have a spaceship traveling to an alien planet 20 light years away from Earth. The relative speed of the Earth and Pilot reference frames is a constant v, such that the time dilation factor $\gamma = 20$.

Now, say you knew length contraction worked like $\Delta x = \alpha \Delta x_0$, where Δx_0 was the proper length, but you didn't know how the factor α depended on v. In that case, you could use consistency to find α. If the Pilot reaches the alien planet, this event will take place in both the Pilot's and the Earth reference frames. If she ages only 1.0 year during the journey, as we calculated using $\Delta t = \gamma \Delta t_0$, she will also have to age exactly 1.0 year if we calculate the journey time using the contracted length in the Pilot's reference frame.

Follow this reasoning and *derive* the correct equation for length contraction. (Find α.)

Answer: In the Earth reference frame, the distance between the planets, 20 light years, is the proper length Δx_0. This is because the length is measured when the Earth and the alien planet is *stationary* in the Earth frame. Therefore, in the Pilot's reference frame, the distance to be traveled will be the contracted length Δx.

In class, we had already calculated that with a large γ, $v \approx c$. We had also established that the time it takes to get to the alien planet is

$$\Delta t = \frac{\Delta x_0}{v} = \frac{20\, c \cdot \text{years}}{c} = 20 \text{ years}$$

$$\Delta t_0 = \frac{\Delta t}{\gamma} = \frac{20 \text{ years}}{20} = 1 \text{ year}$$

Now, we just have to realize that in the Pilot's reference frame,

$$\Delta t_0 = \frac{\Delta x}{v}$$

Using $\Delta x = \alpha \Delta x_0$, we get

$$\Delta t_0 = \frac{\Delta x}{v} = \alpha \frac{\Delta x_0}{v} = \alpha \Delta t$$

Since time dilation gives us $\Delta t = \gamma \Delta t_0$, this means that

$$\alpha = \frac{1}{\gamma}$$

which is what we need for length contraction. In the Pilot's reference frame, she takes only one year to get to the alien planet because to her, the alien planet is rushing toward her at $v \approx c$, starting from a distance of only one light-year away.