Exam 3; Phys 186 Name ______________

Feel free to ask me questions, including whether you've correctly solved any given problem. I am asking you to do some complex reasoning—there is nothing wrong in consulting me.

1. (40 points) You have a spaceship traveling directly away from Earth, with a relative velocity v. Ground control communicates with the spaceship by radio. At some point in the communication, ground control sends out a radio wave with wavelength λ_s and period T_s (s for "source") as measured in the Earth's frame of reference.

(a) Since the ship is moving away, when ground control observes T', the time interval between wave crests reaching the ship, they will not measure $T' = T_s$—that would be true only for a ship that was not moving. Which of the following is the correct expression for T'?

Hint 1: This is an ordinary wave question. It has nothing to do with relativity, since all time intervals in the question are those measured in a single frame of reference, that of ground control. If the question was about water waves going toward a boat moving away from the source of the waves, the answer would be the same.

Hint 2: Even if you don't know what you're doing, only one of the following answers makes any sense whatsoever. Ask yourself: When $v = 0$, what should T' be?

i. $T' = \sqrt{vT_s}$

ii. $T' = \frac{T_s}{1-v/c}$

iii. $T' = \frac{v}{c}T_s$

iv. $T' = (c - v)T_s$

v. $T' = \sqrt{1 - v^2T_s}$

Now explain why the answer you chose is physically correct. (Don't just tell me that all other options are obviously wrong, though this is in fact the case.)

(b) Now let's bring in special relativity. T' is the period of waves reaching the ship from the point of view of an observer on Earth. Given time dilation, what is T_o, the period of the waves in the frame of reference of the observer on the spaceship? *Hint:* Be careful. Which is the frame of reference for which the events of the wave crests reaching the ship happen at the same location? The s frame (Earth) or the o frame (the ship)?

Using your result for T_o, find an equation that relates frequencies rather than periods: find an equation expressing f_o, the frequency observed on the spaceship, in terms of f_s, the source frequency, v, the relative speed, and c, the speed of light.

Mathematical hint: Your end result will be simpler if you use the fact that

$$1 - (v/c)^2 = (1 - v/c)(1 + v/c)$$

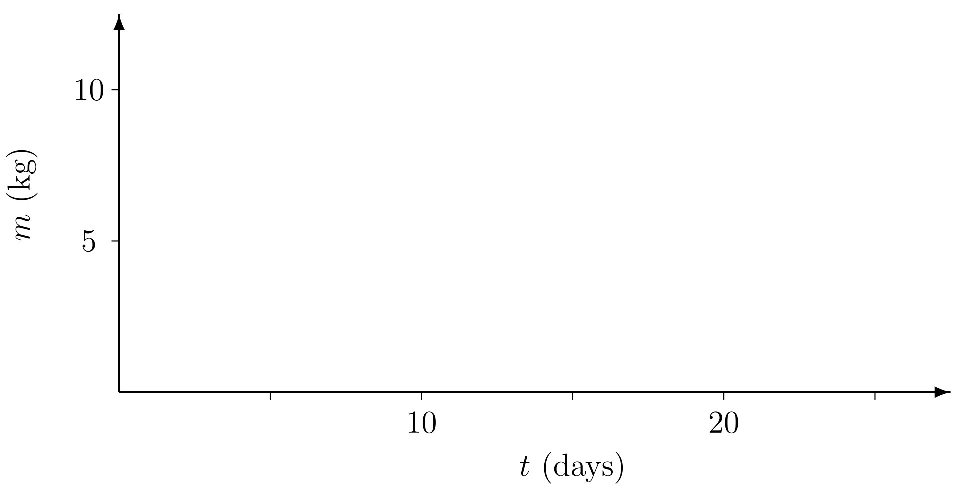
(c) Say you're an astrophysicist looking at light from a distant galaxy. You put the light through a diffraction grating, and find a pattern of spectral lines like that Hydrogen produces on a lab on Earth. But there is a difference: all the lines are shifted to a lower frequency, such that $f_o = \frac{1}{2} f_s$. The lines are, in other words, *redshifted*. (f_o is the observed frequency of each spectral line, and f_s is the source frequency of the line, as known from Hydrogen in the lab.)

Find the ratio v/c, where v is the relative speed of the far away galaxy. Is it moving toward Earth or away from Earth?

2. (40 points) Let's do half-lives.

(a) You have friend who is not a science major. She tells you that quantum mechanical events cannot be truly random. After all, randomness implies unpredictability, but physicists make precise predictions using quantum mechanics. Given the half-life, they can tell you exactly what amount of a radioactive sample will remain after a certain time. Given the energy of photons emitted from a light source, they can calculate the interference pattern observed when a diffraction grating is placed between the source and a screen. Correct your friends' misconceptions and explain what the role of randomness in quantum mechanics is.

(b) You have a 10.0 kg block of radioactive material A, and at time $t = 0$, you start with all 10 kg being pure A. Sketch a graph of the amount of A that remains in the block over time. The half-life of A nuclei is 5.0 days.



(c) Pick, from among the following, the correct expression for the amount of A remaining over time. Here $\tau = t_{1/2}/\ln 2$, and $m_0 = 10$ kg.

i. $m = m_0 \cos \frac{t}{\tau}$

ii. $m = m_0 \left(1 - \frac{t}{\tau}\right)$

iii. $m = m_0 \ln \frac{t}{\tau}$

iv. $m = m_0 \, e^{-\frac{t}{\tau}}$

v. $m = \frac{m_0}{\sqrt{1-(\frac{t}{\tau})^2}}$

(d) What is m at $t = 9.0$ days?

3. (40 points) Recall the experiment we did in the lab, where we produced an electron beam and bent its trajectory by using electric and magnetic fields. Say we accelerated the electrons, starting from rest, by applying a voltage of 4000 V.

(a) Find v, the speed of the electrons after their acceleration is complete, by using the classical expression for kinetic energy. Then calculate the time dilation factor γ for this v, and state whether you think that this result means that you should have used the relativistic kinetic energy instead.

(b) There is a distance L between the point the electrons traveling at v emerge from the electron gun and when they hit the screen. If $L = 0.10$ m, circle what you think is a good estimate for Δx, the uncertainty in the position of the electrons along the direction in which they are traveling.

$$L \qquad \frac{hL}{4\pi} \qquad \frac{h}{4\pi L} \qquad \frac{c}{L} \qquad \frac{\gamma c}{L}$$

Now write down your estimate for the uncertainty: $\Delta x =$ ____________________ m.

(c) Since the electrons are subatomic particles, we have to use quantum mechanics. Since $\Delta x < \infty$, this means that $\Delta p > 0$. In other words, we cannot assume that we know that the electrons are traveling exactly at the speed v you calculated. Estimate Δp, assuming that Δp, your uncertainty about p, is at its *minimum* possible value.

(d) Physicists usually like their electron beams to be "monochromatic"—all particles at a single wavelength. If the value of $\Delta p/p$ is small, the beam is close to monochromatic. Calculate the numerical value of p for your beam of electrons, and then find $\Delta p/p$ and decide whether this beam is almost monochromatic or not.

4. (40 points) You have a circular wire, which you connect to a battery so that a current goes through the loop. You then place this circuit on a table such that the current goes around the loop in a counterclockwise direction when looked at from above. Finally, you take a small superconducting cube, and place it exactly above the center of your current loop. You find it levitates.

(a) Sketch a view of the loop from above, and also show a sideways view with the y-axis as the upward direction. Draw in roughly what the current and the magnetic field looks like. Add a small box to indicate the location in which you will place the superconducting cube.

(b) The currents that arise within the superconductor are such that *within the superconductor*, they create a magnetic field that is exactly equal and opposite to the external field produced by the current loop. Sketch, using appropriate viewpoints, pictures that give a qualitative idea of the direction of the currents circulating within the superconductor once you place it above the center of the current loop.

(c) Now sketch the magnetic forces due to the external magnetic field from the current loop on the currents within the superconductor. This should help you see why the levitation happens. Why does the cube levitate?

(d) Will the cube levitate only at a single height above the loop, or will it levitate at any height you happen to place it? Explain.

5. (40 points) A neutral pion, π^0, is a bound state of an up quark, u, and an anti-up quark $\bar{u}$. The u and $\bar{u}$ are antiparticles of each other: they have the same mass and spin, but opposite charges. The π^0 has a mass of 135 MeV/c^2, and an electron has a mass of 0.511 MeV/c^2.

(a) We want to create a π^0 by colliding an electron and a positron at high speeds, so that $e^- + e^+ \to \pi^0$. (A positron and electron are antiparticles of each other.) This is possible *if* all conservation laws are obeyed. You have learned about the conservation of linear momentum, angular momentum, energy, and charge in this course.

Angular momentum is the hardest for you to check, so I'll do it for you: π^0 has spin 0, therefore if the electron that meets the positron have opposite spins, angular momentum will conserved. So angular momentum conservation does not prevent $e^- + e^+ \to \pi^0$.

Check if linear momentum, energy, and charge conservation prevent $e^- + e^+ \to \pi^0$ or not.

(b) Say you try to isolate a u quark by removing it from the $\bar{u}$ in a π^0, just like you could obtain an isolated electron by providing enough energy to ionize a Hydrogen atom. You find that this does not happen. Instead, if you add an amount of energy E to a pion, you might get a result like

$$\pi^0 + E \rightarrow \pi^0 + \pi^0 \qquad \text{or} \qquad \pi^0 + E \rightarrow \pi^0 + e^- + e^+$$

So you might end up with multiple pions and other particles, but no matter how large E is, you will not isolate a quark. Explain why this is so, using the equivalence of mass and energy ($E = mc^2$) and the fact that the strong force between quarks grows larger as the distance between quarks increases.