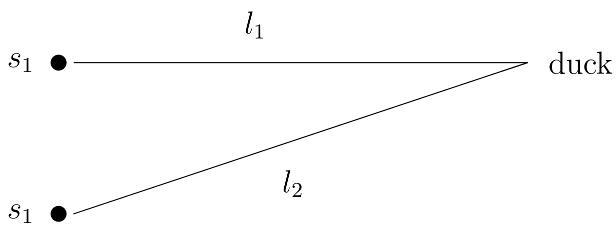
**1. (30 points)** You have two sources of circular waves on the surface of a lake, and a rubber duck floating on the surface, as shown. The distance between the sources is 1.0 m, and the duck is directly in front of the first source.




(a) Say both the sources are in phase, and produce waves with a wavelength of 0.7 m. Sketch a graph that *qualitatively* shows how the intensity of the waves at the location of the duck varies with respect to $l_1$.

**Answer:** The amplitude of each wave from the sourrces decreases as the duck moves farther. On top of that, there is interference. Destructive interference will never give zero total amplitude, since $l_2 > l_1$ always and hence the amplitudes are never equal. Therefore the intensity should never be zero or negative, but should wave up and down as constructive and destructive interference will happen as $l_1$ increases. Overall the intensity should trend downward as you go farther away from the sources.

(b) Say $s_1$ keeps producing waves with $\lambda_1 = 0.7$ m, but $s_2$ now begins to produce waves with $\lambda_2 = 2.6$ m. Would you still get a stable interference pattern on the surface of the lake? Explain.

**Answer:** You would no longer get a consistent interference pattern, just choppiness. Interference requires wavelengths that are are equal or very close to one another.

**2. (30 points)** The $\alpha$-emitting isotope we used in our Nuclear Radiation lab is Po-210, with a half-life of 138.4 days.

(a) Let's say our isotope samples become useless for lab work once their activity declines to 5.0 times the background activity. The background activity is 0.45 counts/second. Say I were to open a new shipment of Po-210 samples right now, and measure their unshielded activity in a typical lab setting to be 68.0 counts/second. For how many days could I use these samples in the Nuclear Radiation experiments?

**Answer:** Activity is proportional to population. Therefore, the ratio of final to initial nuclei populations is

$$\frac{A}{A_0} = \frac{5 \cdot 0.45}{68.0} = \frac{N}{N_0} = 2^{-t/t_{1/2}} = e^{-(\ln 2)t/t_{1/2}}$$

Solving for $t$, we get

$$t = -\frac{\ln 0.0331}{\ln 2} t_{1/2} = 680.6 \text{ days}$$

(b) Is it possible for an individual Po-210 nucleus to survive without decaying for longer than 5 half-lives? Explain.

**Answer:** Yes. *Individual* quantum events, such as radioactive decays, are random. The half-life gives what happens *on average.* And an individual nucleus may well survive for lots and lots of half-lives.

**3. (50 points)** Say you have a Hydrogen atom with the electron in its lowest energy state.

(a) Sketch a picture of the probability of the location of the electron. The proton is at the origin. Indicate probabilities by shading, and briefly explain your picture. (This should be in your class notes, but still show this to me; you need to get it right for the rest of the question.)

**Answer:** The probability will be maximum at the origin, while becoming smaller as we move radially away from the origin. There should be no angle (orientation) dependence, because nothing about the interaction between the electron and proton depends on orientation.

(b) Given your sketch, when you make a measurement of the location of the electron, *on average*, where will it be found? Somewhere along the $\pm x$-axis? The $\pm y$-axis? The origin? Explain.

**Answer:** The origin, because the probability distribution is symmetric—it favors neither $+x$ or $-x$ etc.

(c) Hydrogen will act as an electric dipole if the average location of the negative charge (the electron) is separated in space from the location of the positive charge (the proton). Is Hydrogen in its lowest energy state an electric dipole?

**Answer:** Since the average negative charge is *not* separated from the positive charge, H is not a dipole.

(d) What is the angular momentum due to the orbital rotation of the electron in this lowest energy state? (Consult section 29.5.)

**Answer:** The lowest energy state has $n = 1$ and $l = 0$. Since $l = 0$, the orbital angular momentum is zero. The electron does not rotate!

(e) Atoms can be magnetic dipoles if they have rotating charges, which means current. Is Hydrogen in its lowest energy state a dipole magnet?

**Answer:** Since the electron does not rotate, H is not a magnetic dipole.

(f) Given all this, you should expect atomic Hydrogen to be very hard to interact with. Explain why. (In reality, atomic Hydrogen is very reactive. This goes to show that the physics of covalent bonds is beyond what we learned about this semester.)

**Answer:** The electrical and magnetic interactions involving H are due to higher-order poles than dipoles, and therefore are even weaker. (This ignores some effects due to electron and proton intrinsic angular momenta called spins, but those are also weak.) Therefore, electromagnetic interactions with H should be very weak. We need the more detailed quantum mechanics behind covalent bonds to understand how H is as reactive as it is.

**4. (60 points)** You have a particle confined to 1D-box with length $L$, as described in section 28.5. The wavefunction $\psi(x)$ describing the particle in the lowest energy state is that corresponding to a standing wave with mode number 1 (see page 506).

(a) The probability distribution for the position $x$ of the particles is $\mathcal{P}(x) = |\psi(x)|^2$. Sketch this distribution. (Have me check this—you need to get it right.)

**Answer:** $\mathcal{P}(x)$ will be a half sine-wave squared between $0 \leq x \leq L$, with a peak at $L/2$ and falling off to 0 at the edges of the box. $\mathcal{P}(x) = 0$ everywhere else outside the box.

(b) Say $\Delta x$ is your uncertainty about $x$, which is the standard deviation of the distribution $\mathcal{P}(x)$. Visually, that is the *width* of the distribution. Given your $\mathcal{P}(x)$ graph, make an estimate (it doesn't have to be exact) of $\Delta x$.

**Answer:** $\Delta x \approx L$. The width of the distribution is less than $L$, but that is a good rough estimate..

(c) Low-energy quantum states are typically low in uncertainty. Therefore, assuming that $\Delta x$ and $\Delta p$ are as small as possible while still remaining consistent with the uncertainty principle, estimate $\Delta p$, the uncertainty in momentum.

**Answer:** Minimum uncertainty means $\Delta x \Delta p = h/4\pi$. Therefore

$$\Delta p = \frac{h}{4\pi \Delta x} \approx \frac{h}{4\pi L}$$

(d) Make a rough qualitative graph of the probability distribution of momentum $\mathcal{P}(p)$. Also draw in the width $\Delta p$, the way we did while sketching probability distributions in class. Since the 1D box is stationary, and the particle can't escape, do you expect that positive momenta are more likely than negative? The other way around? Or do you expect positive and negative momenta to be equally probable? Draw your graph accordingly.

**Answer:** Since the particle is confined, it can't have an excess of positive or negative momentum. In fact, $\mathcal{P}(p) = \mathcal{P}(-p)$. The most likely momentum, in fact, is zero. So the sketch should be a symmetric distribution with width $\Delta p$.

(e) Now make a sketch of the probability of the *magnitude* of momentum, $\mathcal{P}(|p|)$. (This is not difficult if you know $\mathcal{P}(p)$. Since $p$ with a given magnitude $|p|$ can be either positive or negative in direction, $\mathcal{P}(|p|) = \mathcal{P}(p) + \mathcal{P}(-p)$.)

**Answer:** $|p| \geq 0$, so the distribution is non-zero only on the positive side of the graph. Adding the positive and negative $p$ probabilities for a symmetric distribution just means that $\mathcal{P}(|p|) = 2\mathcal{P}(p)$ for $p > 0$. The width of the distribution is still about $\Delta p$. If you visually estimate where the average is, $|p|_{avg} \approx \Delta p$.

(f) You should see from the widths of your graphs that a good estimate for the standard deviation of $\mathcal{P}(|p|)$ is also $\Delta p$. Now, the average measured magnitude of momentum, $|p|_{avg}$, can also be estimated as $|p|_{avg} \approx \Delta p$. Draw the rough location of $|p|_{avg}$ on your graph, and also indicate $\Delta p$, showing that they are close.

(g) The particle in a box has only kinetic energy, $K = |p|^2/2m$. Using $|p|_{avg}$, also estimate the energy of the particle in the lowest energy state. Compare this to the exact value given in section 28.5. Is your result acceptable as a rough guess?

**Answer:** Putting everything together,

$$E \approx \frac{(\Delta p)^2}{2m} \approx \frac{h^2}{32\pi^2 L^2 m}$$

The actual value for $n = 1$ is $E_1 = h^2/8L^2m$. This has the right constants, and given all the crude estimations involved, is not bad.

**5. (30 points)** Looking at the highlighted equations (on a yellow background) in your textbook, you would find that while some are fundamental and important, others are much less important. For example, in chapter 10, equation 10.4 expresses energy conservation. This is conceptually fundamental, and it has widespread applicability. I would like you to remember energy conservation after you're done with physics. But equation 10.20 is pointless: it is very specific to perfectly elastic collisions of two bodies, where one starts at rest. I don't care if you remember this for 20 seconds, let alone 20 years.

Look through the highlighted equations in chapter 20 and chapter 25. In each chapter, identify one equation you think is important, and one that is unimportant. Write these down, and briefly explain your decisions.

**Answer:** Individual answers will differ; I am most interested in your reasoning process.