Solutions to Exam 3; Phys 186

1. (30 points) Diving in a pool, you come close to the surface and look at objects outside through a thin, transparent layer of water. You will observe one of the following:

Redshift: The colors will be shifted toward the red end of the visible spectrum: blue colors will become greenish, green yellow, and so on.

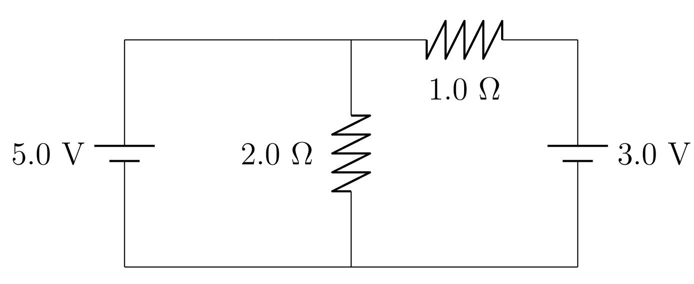
No change: All colors will remain the same.

Blueshift: The colors will be shifted toward the blue end of the visible spectrum: green colors will become bluish, yellow green, and so on.

Knowing that the wavelengths of light waves change upon entering the water from the air, which do you predict? Give your reasons. Note that the molecules in your retina that respond to light absorb a photon and get excited to a higher energy level. So the color you perceive has to do with the *energy* of photons.

Answer: The wavelengths change, but the frequency f does not. Photon energies are proportional to the frequency, $E = hf$, not the wavelength! The photon energies are not affected, and so the color remains the same.

2. (30 points) You have the following circuit. Calculate the voltage across, the current through, and the power dissipated by each resistor.



Answer: There are three currents in this circuit, related by the junction equation $I_0 = I_1 + I_2$. There are two loop equations:

$$5\text{ V} = (2\,\Omega)I_2 \qquad \text{and} \qquad (2\,\Omega)I_2 = (1\,\Omega)I_1 + 3\text{ V}$$

From the first loop equation, $V_2 = 5\,\mathrm{V}$, $I_2 = 2.5\,\mathrm{A}$, and $P_2 = V_2 I_2 = 12.5\,\mathrm{W}$. From the second equation, $V_1 = 2\,\mathrm{V}$, $I_1 = 2\,\mathrm{A}$, and $P_1 = V_1 I_1 = 4\,\mathrm{W}$.

3. (20 points) A physics major tells you that they have a new explanation of black holes. They ask you to imagine a brick, which we heat up to higher and higher temperatures.

- As temperature increases, the atoms making up the brick will jiggle more vigorously.
- Atoms are made of electrically charged particles.
- The atoms will radiate higher amplitude electromagnetic waves.
- More electromagnetic waves in the brick leads to increased destructive interference between the waves;
- Therefore the intensity of the radiation escaping from the brick will decrease.
- Beyond a threshold, *no* radiation will escape: the brick will be completely black.
- We can calculate the threshold using $E = mc^2$. Since mass is equivalent to energy, heating up the brick is equivalent to adding more and more mass.

This reasoning is incorrect. Explain how it goes wrong. Circle the suspicious bullet points.

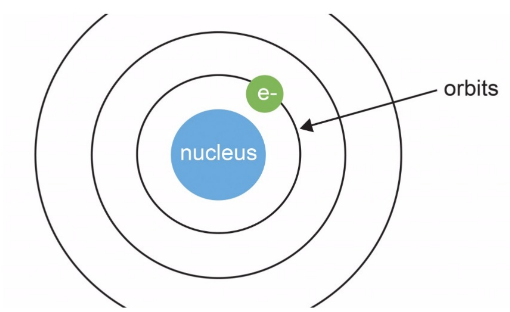
Answer: Things start falling apart with the "increased destructive interference" point. No such thing happens. In fact, there will be an increasing intensity of radiation, proportional to T^4 as we learned in the first semester. The rest is blather; while adding more energy *is* equivalent to adding more mass, the brick will be vaporized long before any measurable increase in mass will take place.

4. (30 points) If you have a glass of water with a mass m_1 and volume V_1, and another with mass m_2 and volume V_2, and you combine them, you get a total mass of $m_1 + m_2$ and a total volume of $V_1 + V_2$.

Let's define the volume of a black hole as the volume within the sphere with the radius r we calculated in class for the event horizon a black hole of mass m. In that case, let's say we have a black hole with a mass m_1 and volume V_1, and another with mass m_2 and volume V_2, and these black holes merge. Will the total mass of the new black hole be less than, equal, or greater than $m_1 + m_2$? Will the total volume be less than, equal, or greater than $V_1 + V_2$?

Answer: For the event horizon, $r \propto m$, while the volume $V \propto r^3$. The merged black home will have mass m_1+m_2—there could be some energy lost to gravitational waves, but this will be negligible compared to the black hole masses. But then, the new radius will be $r_1 + r_2 \propto m_1 + m_2$. Therefore the new volume will be proportional to $(r_1+r_2)^3 > r_1^3+r_2^3$. Therefore $V > V_1+V_2$!

5. (30 points) We briefly discussed an early model for atoms where the electrons did not spiral into the nucleus because only certain circular orbits with fixed radii were allowed. Electrons in these orbits still moved like planets around a star. This is not, however, a proper quantum model, because it violates the uncertainty principle: $\Delta p_x \Delta x \geq h/4\pi$, where the x-axis can be any arbitrary direction in space. Explain exactly how the uncertainty principle is violated for electrons in fixed circular orbits.



Answer: Take the x-axis to be in a radial direction, where the electron is located. In that case, $\Delta x = \Delta r = 0$, since r is fixed. We also know that r is not changing, so therefore the radial momentum $p_x = p_r = 0$. But since the momentum is exactly 0, therefore the uncertainty $\Delta p = 0$. We get $\Delta p \Delta x = 0 < h/4\pi$, violating the uncertainty principle.

A very similar argument could be made with x along the instantaneous direction of movement of the electron in orbit.

6. (40 points) You want to measure the half-life of a radioactive sample. You first measure the background radiation in your lab: 23 counts per minute. Then, every day at 12:00 noon, you take a Geiger counter and count the total number of events with your sample in place for exactly one minute. Call this activity A_T. Here is a table of your data:

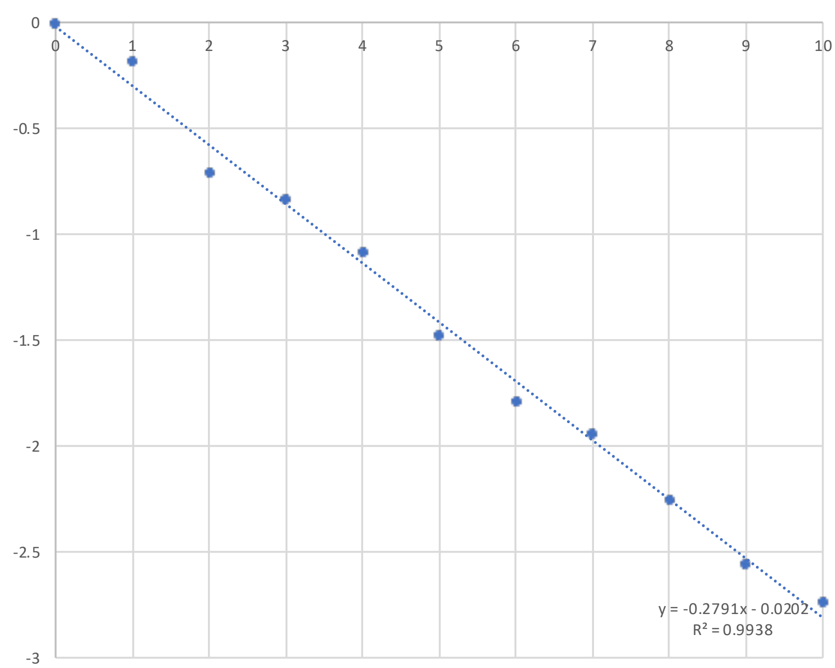
t (days)	0	1	2	3	4	5	6	7	8	9	10
A_T (counts/minute)	123	111	84	79	70	59	52	49	44	40	38

(a) Make a graph involving A_S, *the activity due to your sample alone*, and time. Choose two of the following to put on the axes of your graph:

$$t \qquad 2^{-t} \qquad \log_2 t \qquad t^{-2} \qquad \frac{A_S}{A_{S0}} \qquad 2^{-A_S/A_{S0}} \qquad \log_2\left(\frac{A_S}{A_{S0}}\right) \qquad \left(\frac{A_S}{A_{S0}}\right)^{-2}$$

A_{S0} refers to A_S at day 0. (Math reminder: $2^x = e^{x\ln 2}$ and $\log_2 x = \ln x/\ln 2$.)

Answer: Use t on the horizontal axis and $\log_2\left(\frac{A_S}{A_{S0}}\right)$ on the vertical.



(b) Using the slope of a straight line through your data, calculate the half-life.

Answer: Since

$$A_S = A_{S0}\, 2^{-t/t_{1/2}} \qquad \Rightarrow \qquad \log_2\left(\frac{A_S}{A_{S0}}\right) = \left(-\frac{1}{t_{1/2}}\right) t$$

Therefore

$$t_{1/2} = -\frac{1}{\text{slope}} = 3.6 \text{ days}$$

7. (20 points) We discussed some of the unsolved puzzles in today's physics, including dark matter and dark energy.

(a) We have no idea what practical applications knowledge about dark matter and dark energy would have. What, then, do you think may be some *intellectual* reasons physicists are very interested in solving such a puzzle?

Answer: Answers will vary. But fundamentally, over 95% of the energy in the universe appears to come in forms that we don't know about. Therefore, if we want to know about the basic nature of of our universe, we clearly have to solve this puzzle. We appear to have a very large gap in our knowledge of how the world works, and that is intellectually compelling.

(b) While not a lot of money goes into dark matter and dark energy research, some does, and it is a motivation for some space-based telescope research, which is expensive. Since we can't expect any practical applications, do you think we should continue with such work, or should we devote more resources to curing cancer instead? Explain your reasoning—I'm not looking for a right answer, but I want to see an argument.

Answer: Answers will vary even more.