# Homework Solutions 3 (Schroeder Chapter 3)

## 10

(a) Melting (at constant temperature) increases entropy by

$$\Delta S_a = \frac{Q}{T_i} = \frac{mL}{T_i} = 36.6 \text{ J/K}$$

(b) Integrate for the entropy due the temperature increase:

$$\Delta S_b = C \int_{T_i}^{T_f} \frac{dT}{T} = C \ln \frac{T_f}{T_i} = 11.0 \text{ J/K}$$

(c) The kitchen is at a constant temperature, and the heat transferred to the ice and then water is lost by the kitchen. So the kitchen entropy changes by

$$\Delta S_c = -\frac{mL + mC(T_f - T_i)}{T_f} = -44.1 \text{ J/K}$$

(d) Add all these to get the change in entropy for the universe:

$$\Delta S_{\text{total}} = \Delta S_a + \Delta S_b + \Delta S_c = 3.5 \text{ J/K}$$

This is positive, as it should be.

## 25

(a) Using $\Omega$,

$$S = k \ln \Omega = k \left( q \ln \frac{q+N}{q} + N \ln \frac{q+N}{N} \right)$$

(b) Taking the derivative,

$$\begin{aligned} \frac{1}{T} &= \frac{\partial S}{\partial U} = \frac{1}{\epsilon}\frac{\partial S}{\partial q} = \frac{k}{\epsilon}\frac{\partial}{\partial q}\left[(q+N)\ln(q+N) - q\ln q - N \ln N\right] \\ &= \frac{k}{\epsilon}\left[\ln(q+N) + \frac{q}{q+N} - \ln q - \frac{q}{q} + \frac{N}{q+N}\right] = \frac{k}{\epsilon}\ \ln\left(1 + \frac{N}{q}\right) \end{aligned}$$

Therefore

$$T = \frac{\epsilon}{k \ln(1 + N\epsilon/U)}$$

(c) Inverting,

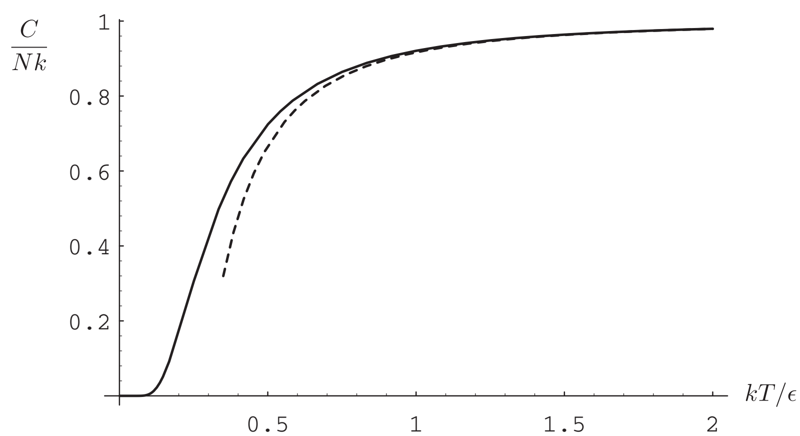
$$U = \frac{N\epsilon}{e^{\epsilon/kT} - 1}$$

$$C = \frac{\partial U}{\partial T} = \frac{N\epsilon^2}{kT^2}\frac{e^{\epsilon/kT}}{(e^{\epsilon/kT} - 1)^2}$$

(d) When $T \to \infty$, $kT \gg \epsilon$ and $e^{\epsilon/kT} \approx 1 + \epsilon/kT$. So we get

$$C \approx \frac{N\epsilon^2}{kT^2}\frac{1}{(\epsilon/kT)^2} = Nk$$

(e) The solid line is the exact result, the dashed line is the approximation in (f).




(f) With $x = \epsilon/kT$,

$$\frac{C}{Nk} \approx \frac{x^2(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3)}{(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 - 1)^2}$$

$$\approx \frac{1 + x + \frac{1}{2}x^2}{1 + x + \frac{7}{12}x^2} \approx 1 - \frac{1}{12}x^2$$

## 34

(a) Since each link is oriented toward the right or the left, this is just like a chain of spins that are up or down. We've done that;

$$S = k \ln \begin{pmatrix} N \\ N_R \end{pmatrix} \approx N \ln N - N_R \ln N_R - (N - N_R) \ln(N - N_R)$$

(b) $L = l(N_R - N_L) = l(2N_R - N)$.

(c) $dU = T\,dS + F\,dL$.

(d) The identity gives

$$F = -T\left(\frac{\partial S}{\partial L}\right)_U \quad \Rightarrow \quad F = \frac{\partial S}{\partial N_R}\frac{\partial N_R}{\partial L} = \frac{\partial S}{\partial N_R}\frac{1}{2l}$$

Using the expressions for $S$ and $L$,

$$F = -\frac{kT}{2l}\ln\left(\frac{N - N_R}{N_R}\right) = -\frac{kT}{2l}\ln\left(\frac{2}{L/Nl + 1} - 1\right)$$

(e) With $L \ll Nl$,

$$\frac{2}{L/Nl + 1} - 1 \approx 1 + \frac{2L}{Nl} \quad \Rightarrow \quad F \approx \frac{kT}{Nl^2}L$$

(f) $F \propto T$. At high temperatures, the links are more randomly oriented, so the chain will tend to shorten.

(g) Adiabatic means no entropy change. A decrease in the $S$ we calculated here, because the configuration becomes less random, will have to be compensated by an increase in the entropy due to the vibration: the temperature will increase.

## 37

(a) The energy of the gas is $U = U_0 + Nmgz$, where $U_0$ is for an ideal gas at $z = 0$. So

$$\mu(z) = \left(\frac{\partial U}{\partial N}\right)_{S,V} = \mu_0 + mgz$$

But $\mu_0 = \partial U_0/\partial N$ is given in the textbook, equation (3.63).

(b) Equilibrium means $\mu_z = \mu_0$. Therefore

$$-kT\ln\left[\frac{V}{N(z)}\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right] + mgz = -kT\ln\left[\frac{V}{N(0)}\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right]$$

Canceling out the terms that appear in both logarithms,

$$kT\ln N(z) + mgz = kT\ln N(0) \quad \Rightarrow \quad N(z) = N(0)\,e^{-mgz/kT}$$