Homework Solutions #1 (McIntyre Chapter 1)

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(a) To normalize, divide by the length of each vector, $\sqrt{\langle\psi|\psi\rangle}$. So the normalized versions of the state vectors are

$$|\psi_1\rangle = \frac{3}{5}|+\rangle + \frac{4}{5}|-\rangle \qquad |\psi_2\rangle = \frac{1}{\sqrt{5}}|+\rangle + \frac{2i}{\sqrt{5}}|-\rangle$$

$$|\psi_3\rangle = \frac{3}{\sqrt{10}}|+\rangle - \frac{e^{i\pi/3}}{\sqrt{10}}|-\rangle$$

(b)

$$\mathcal{P}_{1z+} = |\langle+|\psi_1\rangle|^2 = \left|\frac{3}{5}\right|^2 = \frac{9}{25} \qquad \mathcal{P}_{1z-} = |\langle-|\psi_1\rangle|^2 = \left|\frac{4}{5}\right|^2 = \frac{16}{25}$$

$$\mathcal{P}_{1x+} = |{}_x\langle+|\psi_1\rangle|^2 = \left|\frac{1}{\sqrt{2}}\left(\langle+| + \langle-|\right)\left(\frac{3}{5}|+\rangle + \frac{4}{5}|-\rangle\right)\right|^2 = \frac{49}{50}$$

$$\mathcal{P}_{1x-} = |{}_x\langle-|\psi_1\rangle|^2 = \left|\frac{1}{\sqrt{2}}\left(\langle+| - \langle-|\right)\left(\frac{3}{5}|+\rangle + \frac{4}{5}|-\rangle\right)\right|^2 = \frac{1}{50}$$

$$\mathcal{P}_{1y+} = |{}_y\langle+|\psi_1\rangle|^2 = \left|\frac{1}{\sqrt{2}}\left(\langle+| - i\langle-|\right)\left(\frac{3}{5}|+\rangle + \frac{4}{5}|-\rangle\right)\right|^2 = \frac{1}{2}$$

$$\mathcal{P}_{1y-} = |{}_y\langle-|\psi_1\rangle|^2 = \left|\frac{1}{\sqrt{2}}\left(\langle+| + i\langle-|\right)\left(\frac{3}{5}|+\rangle + \frac{4}{5}|-\rangle\right)\right|^2 = \frac{1}{2}$$

$$\mathcal{P}_{2z+} = |\langle+|\psi_2\rangle|^2 = \left|\frac{1}{\sqrt{5}}\right|^2 = \frac{1}{5} \qquad \mathcal{P}_{2z-} = |\langle-|\psi_2\rangle|^2 = \left|\frac{2i}{\sqrt{5}}\right|^2 = \frac{4}{5}$$

$$\mathcal{P}_{2x+} = |{}_x\langle+|\psi_2\rangle|^2 = \left|\frac{1}{\sqrt{2}}\left(\langle+| + \langle-|\right)\left(\frac{1}{\sqrt{5}}|+\rangle + \frac{2i}{\sqrt{5}}|-\rangle\right)\right|^2 = \frac{1}{2}$$

$$\mathcal{P}_{2x-} = |{}_x\langle-|\psi_2\rangle|^2 = \left|\frac{1}{\sqrt{2}}\left(\langle+| - \langle-|\right)\left(\frac{1}{\sqrt{5}}|+\rangle + \frac{2i}{\sqrt{5}}|-\rangle\right)\right|^2 = \frac{1}{2}$$

$$\mathcal{P}_{2y+} = |{}_y\langle+|\psi_2\rangle|^2 = \left|\frac{1}{\sqrt{2}}\left(\langle+| - i\langle-|\right)\left(\frac{1}{\sqrt{5}}|+\rangle + \frac{2i}{\sqrt{5}}|-\rangle\right)\right|^2 = \frac{9}{10}$$

$$\mathcal{P}_{2y-} = |_y\langle -|\psi_2\rangle|^2 = \left| \frac{1}{\sqrt{2}} \left(\langle +| + i\langle -|\right) \left(\frac{1}{\sqrt{5}}|+\rangle + \frac{2i}{\sqrt{5}}|-\rangle \right) \right|^2 = \frac{1}{10}$$

$$\mathcal{P}_{3z+} = |\langle +|\psi_3\rangle|^2 = \left| \frac{3}{\sqrt{10}} \right|^2 = \frac{9}{10} \qquad \mathcal{P}_{3z-} = |\langle -|\psi_3\rangle|^2 = \left| \frac{e^{i\pi/3}}{\sqrt{10}} \right|^2 = \frac{1}{10}$$

$$\mathcal{P}_{3x+} = |_x\langle +|\psi_3\rangle|^2 = \left| \frac{1}{\sqrt{2}} \left(\langle +| + \langle -|\right) \left(\frac{3}{\sqrt{10}}|+\rangle - \frac{e^{i\pi/3}}{\sqrt{10}}|-\rangle \right) \right|^2 = \frac{7}{20}$$

$$\mathcal{P}_{3x-} = |_x\langle -|\psi_3\rangle|^2 = \left| \frac{1}{\sqrt{2}} \left(\langle +| - \langle -|\right) \left(\frac{3}{\sqrt{10}}|+\rangle - \frac{e^{i\pi/3}}{\sqrt{10}}|-\rangle \right) \right|^2 = \frac{13}{20}$$

$$\mathcal{P}_{3y+} = |_y\langle +|\psi_3\rangle|^2 = \left| \frac{1}{\sqrt{2}} \left(\langle +| - i\langle -|\right) \left(\frac{3}{\sqrt{10}}|+\rangle - \frac{e^{i\pi/3}}{\sqrt{10}}|-\rangle \right) \right|^2 = \frac{1}{2} - \frac{3\sqrt{3}}{20}$$

$$\mathcal{P}_{3y-} = |_y\langle -|\psi_3\rangle|^2 = \left| \frac{1}{\sqrt{2}} \left(\langle +| + i\langle -|\right) \left(\frac{3}{\sqrt{10}}|+\rangle - \frac{e^{i\pi/3}}{\sqrt{10}}|-\rangle \right) \right|^2 = \frac{1}{2} + \frac{3\sqrt{3}}{20}$$

(c)

$$|\psi_1\rangle \doteq \frac{1}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix} \qquad |\psi_2\rangle \doteq \frac{1}{\sqrt{5}}\begin{pmatrix} 1 \\ 2i \end{pmatrix} \qquad |\psi_3\rangle \doteq \frac{1}{\sqrt{10}}\begin{pmatrix} 3 \\ -e^{i\pi/3} \end{pmatrix}$$

(d) Same thing with matrices.

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(a)

$$\mathcal{P}_{S_z=+\hbar/2} = \left| \frac{3}{\sqrt{34}} \right|^2 = \frac{9}{34}$$

$$\mathcal{P}_{S_z=-\hbar/2} = \left| i\frac{5}{\sqrt{34}} \right|^2 = \frac{25}{34}$$

(b)

$$\mathcal{P}_{S_x=+\hbar/2} = |_x\langle +|+\rangle|^2 = \frac{1}{2}$$

$$\mathcal{P}_{S_x=-\hbar/2} = |_x\langle +|-\rangle|^2 = \frac{1}{2}$$

15 Using orthonormality, that $\langle a_n | a_m \rangle = \delta_{mn}$,

$$\mathcal{P}_f = |\langle \psi_f | \psi_i \rangle|^2 = \left| \left(\frac{1-i}{\sqrt{3}} \langle a_1 | + \frac{1}{\sqrt{6}} \langle a_2 | + \frac{1}{\sqrt{6}} \langle a_3 | \right) \left(\frac{i}{\sqrt{3}} | a_1 \rangle + \sqrt{\frac{2}{3}} | a_2 \rangle \right) \right|^2$$

$$= \left| \frac{1+i}{3} + \frac{1}{3} \right|^2 = \frac{5}{9}$$