The rate of heat absorption from space at the CMB temperature of 2.7 K is negligible compared to the rate of heat radiated from a body at 310 K. Therefore your rate of heat loss would be \( \frac{dQ}{dt} = -420 \, \text{W} \), the answer to 2.1. Using \( Q = mc\Delta T \) from first year physics,
\[
\frac{dQ}{dt} = mc\frac{dT}{dt}
\]
With a mass of 70 kg for an about average human, this gives a cooling rate of
\[
\frac{dT}{dt} = -1.4 \times 10^{-3} \, \text{K/s}
\]
At that rate, it would take 710 seconds for your body temperature to drop by 1 K. That’s longer than it would take for you to asphyxiate.

We have a first-order differential equation with constant coefficients, which is solved by an exponential:
\[
E = E_0 e^{-kr}
\]
where \( E_0 \) is the initial energy of a photon at \( r = 0 \). A photon’s energy is \( E = hf = hc/\lambda \). Therefore
\[
z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{E_0}{E} - 1
\]
Putting in the exponential distance dependence,
\[
z = \frac{E_0}{E_0 e^{-kr}} - 1 = e^{kr} - 1 \approx 1 + kr - 1 = kr
\]
with the approximation at \( kr \ll 1 \). This is the Hubble behavior, with \( k = H_0/c \). Therefore
\[
k = \frac{H_0}{c} = 2.3 \times 10^{-4} \, 1/\text{Mpc}
\]
When \( hf \ll kT \), \( e^{hf/kT} \approx 1 + hf/kT \), and therefore \( n(f) \approx 8\pi kT f/hc^3 \). Integrate this to get the number density of photons with \( hf < E_0 \):
\[
n \approx \frac{8\pi kT}{hc^3} \int_{0}^{E_0/h} df \frac{f}{h^3 c^3} = \frac{4\pi kT E_0^2}{h^3 c^3}
\]
From the textbook, the total number density of photons is $n_\gamma = \beta T^3$. Using the given value for $\beta$,

$$\frac{n}{n_\gamma} = 0.208 \left( \frac{E_0}{kT} \right)^2$$

$\lambda = 3$ cm means $E_0 = \frac{hc}{\lambda} = 4.133 \times 10^{-5}$ eV. The CMB has $kT = 2.349 \times 10^{-4}$ eV, so that $E_0 / kT = 0.176$. It’s a bit dangerous to rely on our assumption that $E_0 \ll kT$, as this means $0.176 \ll 1$. But this is astrophysics, and such assumptions are not that bad in this context. Therefore

$$\frac{n}{n_\gamma} = 0.006$$