Homework Solutions 1 (Ryden Chapter 2)

3 The rate of heat absorption from space at the CMB temperature of 2.7 K is negligible compared to the rate of heat radiated from a body at 310 K. Therefore your rate of heat loss would be $dQ/dt = -420$ W, the answer to 2.1. Using $Q = mc\Delta T$ from first year physics,

$$\frac{dQ}{dt} = \frac{mc\,dT}{dt}$$

With a mass of 70 kg for an about average human, this gives a cooling rate of

$$\frac{dT}{dt} = -1.4 \times 10^{-3} \text{ K/s}$$

At that rate, it would take 710 seconds for your body temperature to drop by 1 K. That's longer than it would take for you to asphyxiate.

4 We have a first-order differential equation with constant coefficients, which is solved by an exponential:

$$E = E_0 e^{-kr}$$

where E_0 is the initial energy of a photon at $r = 0$. A photon's energy is $E = hf = hc/\lambda$. Therefore

$$z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{E_0}{E} - 1$$

Putting in the exponential distance dependence,

$$z = \frac{E_0}{E_0 e^{-kr}} - 1 = e^{kr} - 1 \approx 1 + kr - 1 = kr$$

with the approximation at $kr \ll 1$. This is the Hubble behavior, with $k = H_0/c$. Therefore

$$k = \frac{H_0}{c} = 2.3 \times 10^{-4} \text{ 1/Mpc}$$

6 When $hf \ll kT$, $e^{hf/kT} \approx 1 + hf/kT$, and therefore $n(f) \approx 8\pi kTf/hc^3$. Integrate this to get the number density of photons with $hf < E_0$:

$$n \approx \frac{8\pi kT}{hc^3} \int_0^{E_0/h} df\, f = \frac{4\pi kT\, E_0^2}{h^3 c^3}$$

From the textbook, the total number density of photons is $n_\gamma = \beta T^3$. Using the given value for β,

$$\frac{n}{n_\gamma} = 0.208 \left(\frac{E_0}{kT} \right)^2$$

$\lambda = 3$ cm means $E_0 = hc/\lambda = 4.133 \times 10^{-5}$ eV. The CMB has $kT = 2.349 \times 10^{-4}$ eV, so that $E_0/kT = 0.176$. It's a bit dangerous to rely on our assumption that $E_0 \ll kT$, as this means $0.176 \ll 1$. But this is astrophysics, and such assumptions are not that bad in this context. Therefore

$$\frac{n}{n_\gamma} = 0.006$$