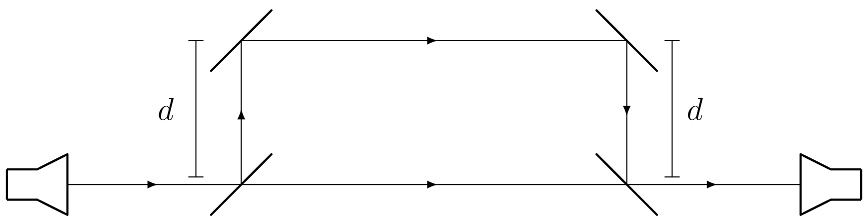
Solutions to Assignment 2; Phys 186

1. (20 points) You have the following interferometer set up in a lab:



You have a microwave source on the left, and a microwave detector on the right. The microwaves have a wavelength of 3.00 cm.The two diagonal lines on top stand for fully silvered mirrors; the two diagonal lines lined up with the source and detector are half-silvered mirrors that reflect and transmit half the incident microwave intensity. The distance between the pairs of fully silvered and half-silvered mirrors is $d = 10.0$ cm. All other distances are irrelevant to the problem.

In other words, you split the microwave beam in two, and make one part travel a longer distance before joining the original beam.

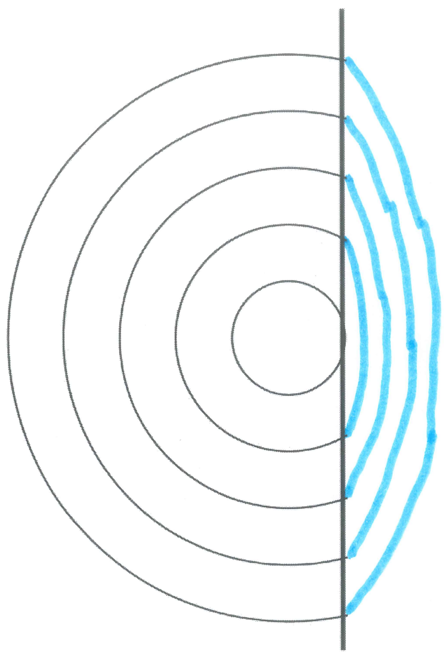
Now you start increasing d. Find the next two values of d for which you get *constructive* interference (an intensity maximum at the detector).

Answer: The path length for the two beams differs by $l_1 - l_2 = 2d$. The condition for constructive interference is $l_1 - l_2 = m\lambda$. At first, $2d$ is not an integer multiple of λ. But if we move the mirrors so that $2d = 21$ cm, this is a multiple of λ. This means $d = 10.5$ cm. The next point of constructive interference will come at $2d = 21 + 3 = 24$ cm, which happens when $d = 12$ cm.

If you really wanted to account for everything, you would also deal with the half-wavelength phase shifts that come with each reflection. But these add up to an integer multiple of a wavelength, and hence do not affect the interference at all.

2. (30 points) You have water waves on the surface of a lake, with wavefronts that spread as concentric circles from a central source, traveling at a constant speed v_{deep}. The straight line indicates a boundary where the lake bottom suddenly steps up, so the waves enter a shallow region where $v_{\text{shallow}} < v_{\text{deep}}$.

(a) Draw how the wavefronts will look in the shallow part of the lake.



(b) Briefly explain your reasoning.

Answer: In the shallow region, the frequency will remain the same, so the wavelength must become smaller. The wavefronts will extend into the shallow region without new wavefronts being created or destroyed at the interface, so each circle will have to be completed.

3. (50 points) You have an electric dipole arranged on the x-axis: a $+q$ charge at $x = +a$ and a $-q$ charge at $x = -a$. The charges are connected by a rigid rod, so the distance between them never changes.

(a) Calculate the electric field created by this dipole on a point on the x-axis, for $x > a$. Get both magnitude and direction.

Answer: At $x > a$, the distance to $+q$ is $x - a$ and the distance to $-q$ is $x + a$. The magnitudes of the electric field due to the two charges are

$$E_+ = \frac{kq}{(x-a)^2} \qquad\qquad E_- = \frac{kq}{(x+a)^2}$$

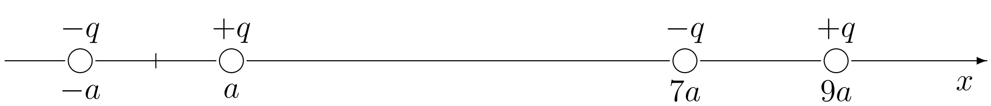
$E_+ > E_-$, because $+q$ is closer to points where $x > a$.

Note that $\vec{E}_+$ points in the $+x$ direction, and $\vec{E}_-$ points in the $-x$ direction. Both electric fields are along the x-axis, with no y component. So we can add them together without worrying about trigonometry. The total electric field is

$$E = E_+ - E_- = kq \left[\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2} \right]$$

The direction of $\vec{E}$ for $x > a$ is in the $+x$ direction.

(b) Calculate the force the first dipole exerts on another dipole further down the x-axis. Get both magnitude and direction.



Answer: Use the electric field found for $x = 7a$ and $x = 9a$, and multiply by the charges at those locations:

$$F_{7a} = kq(-q) \left[\frac{1}{(6a)^2} - \frac{1}{(8a)^2} \right] = -0.012 \, \frac{kq^2}{a^2}$$

$$F_{9a} = kq(+q) \left[\frac{1}{(8a)^2} - \frac{1}{(10a)^2} \right] = 0.0056 \, \frac{kq^2}{a^2}$$

The total force is

$$F = -0.0064 \, \frac{kq^2}{a^2}$$

The $-$ sign indicates a force in the $-x$ direction: the second dipole is attracted by the first.

(c) You now have the second dipole oriented perpendicular to the first:

Qualitatively sketch the forces on this second dipole due to the first dipole. Also indicate in what direction (clockwise or counterclockwise) it rotates. Very briefly explain why.

Answer: The $-q$ on the second dipole will be attracted toward the first dipole, because the $+q$ on the first dipole is closer. The $+q$ on the second dipole will be repelled away from the first dipole, again because the $+q$ on the first dipole is closer. This pair of forces will produce a counterclockwise rotating torque. So the second dipole will tend to align with the first one.

(d) What can you conclude about dipole-dipole interactions from this problem? Do dipoles attract or repel one another? How do they orient themselves relative to each other?

Answer: Dipoles attract, and they orient themselves such that arrows you draw from the $-$ to the $+$ charges in each dipole are parallel to one another.