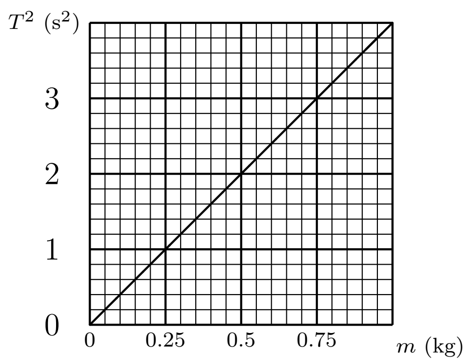
Solutions to Exam 1; Phys 186

1. (50 points)

(a) Imagine you hung masses from the end of an ideal spring, with zero mass and no limits on extension or compression without deformation. The spring constant is $k = 10.0$ N/m. You use masses m varying between 0.00 kg and 1.00 kg, and set each mass oscillating, measuring the period T of the resulting motion. What would your predicted graph for the square of the period (T^2) versus mass (m) look like? Put in appropriate numbers on the axes.



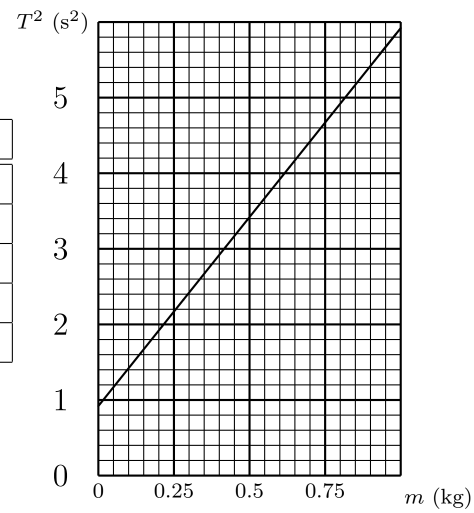
Answer: Since $T = 2\pi\sqrt{m/k}$, its square goes like

$$T^2 = \left(\frac{4\pi^2}{k}\right) m = (3.95\ \mathrm{s}^2/\mathrm{kg})\, m$$

This is a linear dependence, with a line going through the origin.

(b) You now get a real spring, and make the following measurements of period for different masses. Make the graph and find k. (*Hint:* Use the slope.)

m (kg)	T (s)	T^2 (s^2)
0.00	0.99	0.98
0.25	1.49	2.22
0.50	1.86	3.46
0.75	2.17	4.71
1.00	2.43	5.90



Answer: The slope of the graph is $4\pi^2/k$. Therefore, figuring out the slope and solving for k, we get 8.0 N/m.

(c) From your graphs, what is the difference between the behavior of the real and ideal springs? Which of the following equations, with an extra parameter ξ, do you think best describes the real spring's period? (*Hint:* $\xi = 0$ for an ideal spring, $\xi \neq 0$ for a real spring.)

$$T = 2\pi\sqrt{\frac{m+\xi}{k}}$$

Answer: This is the only equation that gives $T^2 \neq 0$ when $m = 0$. The difference between the real and ideal springs is that the real spring graph does not go through the origin—at $m = 0$, with no extra masses hanging, the real spring still behaves as if there is some mass hanging on it.

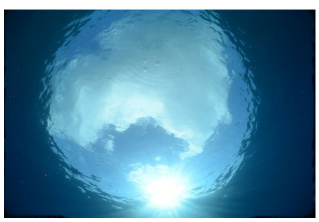
(d) What property of the real spring, ignored in an ideal spring, do you think gives rise to the parameter $\xi \neq 0$? Explain your reasoning.

Answer: The extra mass is due to the mass of the spring itself, which is neglected for an ideal spring.

2. (30 points) Say you have a diffraction grating with with slit spacings just slightly larger than visible light wavelengths. When you shine a narrow beam of white light through this grating onto a screen, you will see a bright white spot straight down the middle, and after some darkness, a rainbow pattern on either side. Will the rainbow pattern have red light closest to the white in the middle, with violet farthest away, or violet closest to the middle, with red farthest away? Explain, using the appropriate equations.

Answer:

3. (30 points) If you dive down under water on a bright sunny day and you look up, you will see the whole sky within a circle on top of you, with darkness on the sides.



(See https://tinyurl.com/unaqqtu for a color version.)

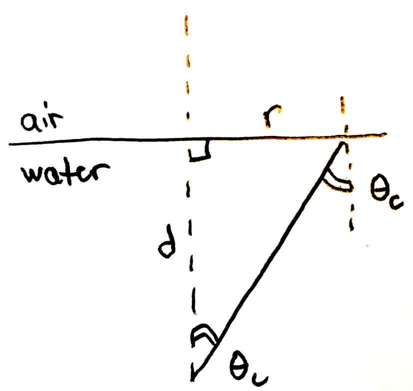
(a) Explain what is happening. (Looking up an online explanation is fine, but make sure what you write here is *only* in terms of what we discussed in class.)

Answer: Since the index of refraction of water is larger than that of air, the direction of propagation of light waves will bend closer to the vertical once they enter the water. All of the sky means an entrance angle θ_1 of 0 to 90° with the vertical. In the water, the direction of propagation will go as $\theta_2 = \sin^{-1}(\sin\theta_1/n_2)$, from 0 to a critical angle $\theta_c < 90°$. So all the light from the sky will come down in a cone defined by θ_c, and you will see a circle of light above.

(b) Let's say you're at a depth d below the surface. Express the radius r of the circle of light in an equation that depends on d and anything else you think is necessary.

Answer: If the light is coming down in a cone with angle $\theta_c < 90°$, the relationship of r to d will be that $r/d = \tan\theta_c$. Looking up the index of refraction of water, $n_2 = 1.333$, we get

$$r = \left(\tan \sin^{-1} \frac{1}{1.333}\right) d = 1.13\, d$$



4. (60 points) You have three charges: a $+q$ charge fixed at $x = 0$, $y = a$, another $+q$ charge at $x = -b$, $y = 0$, and a $-2q$ charge at $x = 0$, $y = 0$.

(a) Find an expression for the total electric field created by these charges on a point on the x-axis, for $x > 0$. (Find the x- and y-components of this electric field.)

Answer: This charge configuration is a combination of a dipole on the x-axis, solved in your second homework assignment, and a dipole on the y-axis, solved in your first practice set of problems. So we can add those two solutions. The x-axis dipole is oriented opposite to the homework problem, so its electric field will be the negative of the known solutions. Its charges are separated by b rather than $2a$, so we have to replace a with $b/2$. Also, its center is shifted so that we have to replace x with $x + b/2$. With all those fixes, we have

$$E_{1x} = kq\left[-\frac{1}{x^2} + \frac{1}{(x+b)^2}\right] \qquad E_{1y} = 0$$

The y-axis dipole is identical to that in the practice set, so we just use that result directly:

$$E_{2x} = kq\left[\frac{x}{(x^2+a^2)^{3/2}} - \frac{1}{x^2}\right] \qquad E_{2y} = -\frac{kqa}{(x^2+a^2)^{3/2}}$$

The total electric field is

$$E_x = kq\left[\frac{x}{(x^2+a^2)^{3/2}} - \frac{2}{x^2} + \frac{1}{(x+b)^2}\right] \qquad E_y = -\frac{kqa}{(x^2+a^2)^{3/2}}$$

(b) Find the voltage due to these charges at a point on the x-axis, for $x > 0$.

Answer: Add the voltages due to all three charges—the only thing that differs is the various distances to a point along the x-axis:

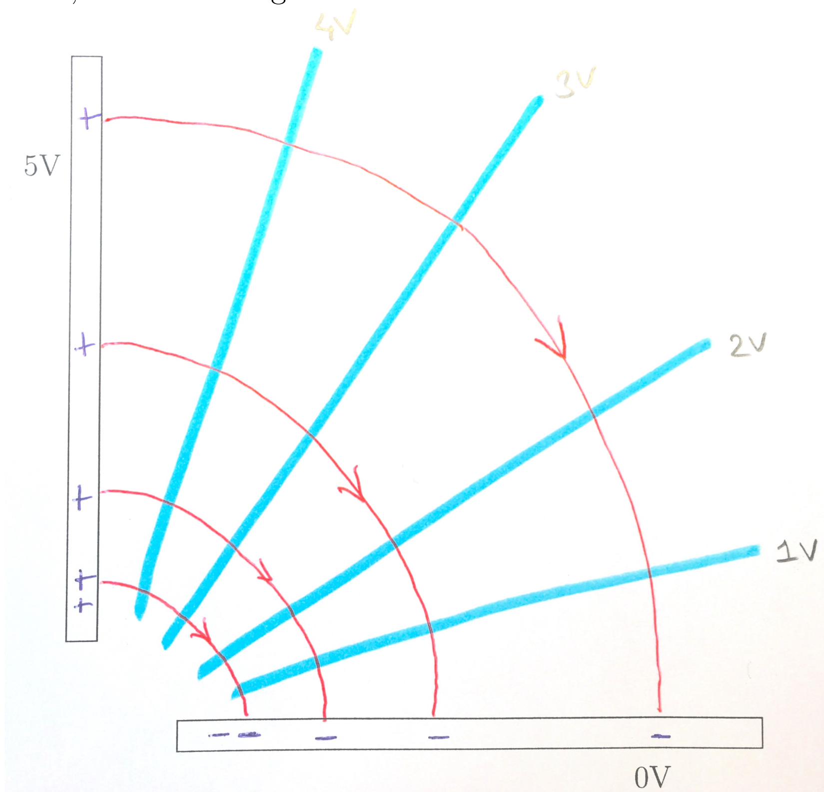
$$V = kq\left[\frac{1}{\sqrt{x^2+a^2}} - \frac{2}{x} + \frac{1}{x+b}\right]$$

(c) Let's say $q = 1.0$ nC, $a = 1.0$ nm, and $b = 2.0$ nm. You put a charge $Q = -2.0$ nC at location $x = 2.0$ nm, $y = 0$. What is an expression for the energy you have to add (or take away) from this charge to bring it to a location $x = 4.0$ nm, $y = 0$?

Answer: Energy is charge times voltage, so

$$\Delta U_E = (-Q)\Delta V = -(8.99\times10^9 \mathrm{Nm}^2\mathrm{C}^{-2})(2\text{ nC})(1\text{ nC})[\ldots] = -3.81\text{ J}$$

5. (30 points) In your equipotential lines lab, say you set up two of the metal plates as shown. You set the voltage of the plate drawn vertically here to 5.0 V, and the voltage of the horizontal one to 0.0 V.



(a) On the diagram above, draw what you think the equipotential lines will look like, with lines for 1V, 2V, 3V, 4V. You need not be concerned about any place other than the space between the plates. (Make sure I look at this and tell you that you've got this roughly right before moving on to the other parts.)

(b) Now add the electric field lines.

(c) There will be an excess of + charges on the 5V plate, and an excess of − charges on the 0V plate. Draw in some + and − charges on each of these plates in a way that you think would produce the field lines that you drew. (*Hint:* the charges will *not* be equally spaced!)