Comments on Ryden Chapter 8

We had stopped after talking about section 8.2 in class. Here's what follows. (Note that much of chapter 8 is technical, and is best followed by reading the book. I'll only list highlights and make comments where I think is necessary.)

8.3

The discussion of recombination here is primarily a long statistical mechanical problem. For those who have already had their statistical mechanics course, this should, I hope, be possible to follow without too much difficulty; if not, you've developed rust. For those who haven't had Phys 486, I'll assume you're at least at the level you saw in Modern II. When reading physics on your own, you will often run into material that is more advanced, or just outside your acquaintance. It's a useful skill to see how much of the argument you can follow nonetheless, making note of whether you'll have to go back and learn more about the topic, or if you can take it for granted and still understand the cosmology-related parts.

The assumption made is that the particles concerned are in equilibrium—thermal, kinetic, and chemical equilibrium. (Reminder from Phys 486: you encountered this as diffusive equilibrium, requiring introduction of a chemical potential μ, which you can roughly think of as an energy cost for adding new particles.) Strictly speaking, an expanding universe is not in equilibrium; fast expansion in particular can drive the system far away from equilibrium. In some contexts this is important—the fact that the entropy of the universe increases slower than the maximum possible entropy can drive local formation of order, since the universe is driven farther away from "heat death." But for recombination, what we're concerned about is the gas of H, γ, p, and e^-; that is close enough to equilibrium to do standard statistical mechanics.

You will notice, in the equations in this section, factors of

$$\bar{n}_{\mathrm{BE}} = \frac{1}{e^{(E-\mu)/kT} - 1} \qquad \text{and} \qquad \bar{n}_{\mathrm{FD}} = \frac{1}{e^{(E-\mu)/kT} + 1}$$

These are the average occupancies (number of particles) for each energy level E, for Bose-Einstein and Fermi-Dirac statistics—fermions and bosons. All of you should at least recognize that. Since photons are massless and can have arbitrarily low energy, the cost of adding new photons is zero, so for photons, the chemical potential $\mu = 0$.

The rest of this section consists mainly of technical details, working out the calculation. Do at least read through it, and if it seems completely foreign to you, ask me questions. Table 8.1 summarizes the important results.

8.4

The temperature fluctuations have a strong dipole component. The standard interpretation of this is that it is due to a local "peculiar velocity" relative to the comoving frame of reference where the CMB is isotropic. Or there might be a "non-Hubble flow" at cosmological scales that leads to the dipole. Noah will be looking into this for his project.

In any case, there is a lot of information in the temperature fluctuations, and we usually analyze these by doing a multipole expansion. Those of you who have taken E& M or Quantum have encountered "spherical harmonics" before. If you haven't, there's no reason to be intimidated. Any angular dependence in 3D can be expanded in terms of a set of basis functions called spherical harmonics, just like any 1D wave can be expanded as a Fourier series where the basis functions are sine waves.

$$f(\theta,\phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} f_{lm}\, Y_l^m(\theta,\phi)$$

Here, $l = 0$ is the monopole component, $l = 1$ the dipole, and so forth. The f_{lm} are constants, analogous to Fourier components.

8.5

The temperature fluctuations are due to fluctuations in density, and hence gravity. Photons climbing out of local gravity wells have less energy—they're redshifted. If they're going down a hill, they're blueshifted.

This is why the CMB is studied closely to get an idea about the causes of structure formation, inflation, and so forth. Small fluctuations in density, even if it's originally due to something like quantum effects in the epoch of inflation, will grow over time. After all, gravity is a purely attractive force, and density variations therefore produce instabilities (growth of unevenness with positive feedback).

Cosmologists are very interested in the acoustic oscillations found in the CMB data. This is discussed in the final few pages. Read it, but it's technical; I won't expect you to fully grasp it. Figure 8.7 is a classic, though.

Recitation and Homework

If you want to do a recitation problem, 8.2 is suitable.

For homework, just 8.4. It may be best to combine this with the problems from chapter 9, so I won't set a deadline yet.