Comments on Ryden Chapter 10

Some of this chapter is technical, but less so than chapters 8 and 9. The main point is that even with its impressive success in explaining things like the relative abundance of chemical elements, the basic Hot Big Bang model also brings up some puzzles. Inflation, which is a period of exponential, faster-than-light expansion of the universe, solves these puzzles, though it also introduces others. (There’s physics for you. The work is never done.)

10.1

The first puzzle is why the universe seems so flat today, since the energy density has to be very close to the critical density for near-flatness, and there doesn’t seem to be any physical reason why this should be the case. Mind you, there also isn’t any physical reason why $\Omega_0 \approx 1$ shouldn’t happen.

If you think of $\Omega_0$ as a parameter in a cosmological model that you fit to the data, there isn’t any great problem. Every theory has free parameters. For example, so far in your physics experience, you’ve been used to treating the masses of particles such as electrons and quarks as numbers you look up—these are experimentally determined rather than calculated by any more fundamental theory. Now, you can certainly stand back and ask why we have the mass values that we observe. After all, masses seem to be peculiarly free parameters. Elementary particle charges, for example, seem much more restricted: every charge comes in integer multiples of $e/3$ (remember that quark and antiquark charges are $\pm e/3$ and $\pm 2e/3$). Masses don’t behave like that. But then again, mass is energy, and in quantum mechanics we’re used to calculating energy levels, as in the $H$ atom. So perhaps, if we ever figure out quantum gravity, we will also be able to calculate quark masses. Or, even with quantum gravity, the situation may be more complicated. Some of the known mechanisms for acquiring mass, such as the Higgs mechanism for electrons, rely on spontaneous symmetry breaking. Broken symmetries shift energy levels, but the results of symmetry breaking are random.

In other words, free parameters in a theory may suggest more fundamental physics we haven’t figured out yet, quite possibly combined with whatever chance throws up. (Randomness in basic physics shouldn’t bother you, once you’re used to quantum mechanics.)

In the case of $\Omega_0$, though, the present constraint of $|1 - \Omega_0| \leq 0.005$ has
bothered many cosmologists more than the usual slight annoyance of having a free parameter. (And remember, there are always free parameters.) It’s mainly the intuition that $\Omega_0 \approx 1$ is overly “fine-tuned” to a value that seems special. It’s not much more than an intuition, since in the absence of a more fundamental theory we have no idea of what a probability distribution for a completely free parameter $\Omega_0$ might be. Therefore we can’t formally argue that flatness is improbable. Nonetheless, this sort of thing bothers physicists, and also excites us because it might be a clue about some more fundamental physics.

Now, we’ve looked at cosmological models in earlier chapters, and depending on what kind of energy dominates, we can trace whether the overall energy density will diverge from the critical value or converge toward it. Writing the Friedmann equation in terms of $\Omega$, we have, in a slight variation of equations 10.1, 2, and 4 in the book,

$$1 - \Omega = -\kappa \frac{c^2}{R_0^2} \frac{1}{\dot{a}^2} = H_0^2 (1 - \Omega_0) \frac{1}{\dot{a}^2}$$

In other words, $1 - \Omega$ is inversely proportional to $\dot{a}^2$. In radiation and matter-dominated universes, we have seen in chapter 5 that $\dot{a}$ behaves like a negative power of $t$, which means that $\Omega$ should diverge from 1: In the early universe, $\Omega$ must have been much closer to 1 than what we see right now (which is compatible with exactly 1). In a dark energy-dominated universe, such as that where $\Lambda$ is most important, the expansion accelerates, so $\dot{a}$ grows with time. In such universes, $\Omega$ will driven towards 1.

So, if fine-tuning bothers you, you have a number of options. You might explore a mechanism to drive $\Omega$ toward 1 in the early universe— inflation. You might explore multiple-universe theories, where lots of bubble universes get produced, and all end up with different physical parameters as a result of symmetry breaking, and we just happen to be in a universe where $\Omega_0 \approx 1$. Some more advanced inflationary cosmologies do a bit of both. Or you can come up with other ideas. Remember, though: the point is not just to come up with a scenario that superficially explains the fine-tuning. That’s easy to do. The difficult part is to come up with a scenario that has independent evidence in support of it, maybe solves multiple independent puzzles, and that doesn’t introduce too many new free parameters of its own. Inflation is a good idea, with much in favor of it, but no variant of inflation is fully satisfactory on all counts yet.
10.2

Another, more serious puzzle with the standard Hot Big bang model is the “horizon problem,” which is about how everything we observe in the universe seems the same at cosmological scales, even though they’re far enough apart to be causally disconnected.

The horizon distance in our universe is not $\infty$. If we look at the CMB from opposite points of the sky, the separation between is larger than $d_{\text{hor}}$, and therefore no light—no physical influence—could have been exchanged between these points. And yet, we have seen that the CMB is remarkably uniform. Even at large angular separations, the temperature fluctuations hover around one part in $10^5$. This suggest thermal equilibrium between all points in our observable universe. But if some of the parts of the universe are causally disconnected from one another, they can’t get to any thermal equilibrium.

In that case, we have to say that what we observe was once causally connected, so it could reach thermal equilibrium, but then everything was separated so that they now have distances farther than light could travel in the age of the universe. In other words, they have to be separated at speeds exceeding the speed of light.

But then, we have already seen, earlier in the semester, that this can happen. No *signal* can travel faster than light, but the universe can expand so fast that points within can be separated at a rate faster than the speed of light. Inflation proposes a brief period in the very early universe where the universe expanded much faster than light, taking a very small causally connected volume of space and inflating it into what became our observable universe today.

10.3

The third puzzle is the lack of magnetic monopoles in our universe. This is the least serious puzzle, because its details depend on the present program of unifying fundamental forces, which is somewhat bogged down at the moment.

Magnetic monopoles are the magnetic analogue of electric charges. Maxwell’s equations account for electric charges and currents, for example in Gauss’s law, $\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$, but the magnetic equivalent is $\vec{\nabla} \cdot \vec{B} = 0$. A more symmetric expression would have been $\vec{\nabla} \cdot \vec{B} = \mu_0 \rho_m$, where a magnetic
monopole density $\rho_m$ shows up. But as the established form of Maxwell’s equations indicate, we don’t see any magnetic monopole around us. It turns out that the spontaneous symmetry-breaking process in some Grand Unified Theories (GUTs) can produce magnetic monopoles. This needs some explanation.

First, spontaneous symmetry breaking. This happens when the expression for the total energy (Hamiltonian) of a system is invariant under a symmetry transformation, but the lower energy states of the system are not invariant. (In quantum terms, the ground state is degenerate, and each ground state has lower symmetry than the Hamiltonian. In statistical mechanical terms, replace “Hamiltonian” with “partition function.”) In such cases, which are common, the high-energy (equivalently, high-temperature) state of the system will be highly symmetric, but as it cools down, the system will evolve toward a less symmetric state, and which of the less symmetric states you end up in will be random.

In statistical mechanics, you may recognize this change in symmetry during cooling as a kind of phase transition. For example, in a ferromagnet, neighboring spins interact such that if they’re aligned, the energy is lower. But the overall Hamiltonian (or partition function) is invariant under spatial rotations. At high temperatures, the spin orientations will be random, and the state of the system will not single out any orientation in space. But as you cool the system down, the preference for aligned spins will take over, and below a transition temperature, there will be an overall magnetization, picking out an arbitrary direction in space.

When unifying fundamental forces, one question is why, at our low energies, we see very different forces: the strong force, weak force, and electromagnetism. Unification proposes that at high energies, the fundamental symmetry behind all such interactions is restored, and all the forces behave under a unified description. But at lower energies, due to spontaneous symmetry breaking, our universe ends up in a random ground state where the forces are very different from one another.

With GUTs, we expect to see such a process. Causally disconnected regions of space, however, will cool down into different ground states. Similar phenomena are common; real ferromagnets don’t cool down to give an overall bulk magnetization. Instead, they form intermediate-scale domains with different spin orientations. Domain boundaries are “topological defects,” which come in various forms and would also be seen in a cooling universe. Point defects in GUTs would be magnetic monopoles.
The problem is, such a process would produce a high density of magnetic monopoles (see the calculation in the textbook), and we don’t see any at all. So we want inflation to immensely dilute the concentration of monopoles in the universe.

We have some indirect evidence that monopoles should exist: if even a single monopole existed, it would explain quantization of electric charge in integer multiples of $e/3$. We have less evidence that any GUT is correct. The most straightforward versions of GUT have already been disconfirmed, since they predicted proton decay which, though it would have been at a very slow rate, could have been observable but has not been observed.

10.4

Inflation proposes a form of dark energy that dominates a brief very early period of the universe. There have, naturally, been attempts to construct models where the dark energy we think we have now, and the dark energy during inflation are the same, but most inflationary models have different dark energies, since a cosmological constant has a constant energy density, while the energy density for the dark energy responsible for inflation has to be much higher than our current dark energy. The simplest, early forms of inflation just proposed a scalar field, whose field quantum was a spin-0 particle, maybe even something already postulated like the Higgs. The simpler models soon ran into difficulties, and inflationary cosmologies today are full of complicated models with otherwise unknown inflaton fields and all sorts of effectively free parameters, causing much tearing out of hairs among theorists.

In any case, if what we had was an early-universe version of a dominant cosmological constant, $\Lambda_i$, we would get exponential growth in the scale factor, as found in chapter 5:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda_i}{3} \quad \Rightarrow \quad a \propto e^{H_i t} \text{ with } H_i = \sqrt{\frac{\Lambda_i}{3}}$$

Following the rough calculation of the book, if the inflationary period lasts for $\Delta t$, the universe will inflate to $e^N$ its original size, with $N = H_i \Delta t$. The time $\Delta t$ is very short, so we need a very large $H_i$. Note that with rapid exponential inflation, the speed by which originally causally connected points in the universe separate will be well over the speed of light.
With accelerated expansion, as always, we will drive $\Omega \to 1$, even if $\Omega$ started out at a fairly random value. Think of this like a balloon inflating exponentially rapidly. The curvature of the spherical surface will end up looking very very flat, even if the original curvature was very pronounced. Inflation, therefore, solves concerns about fine-tuning.

$$|1 - \Omega| \propto e^{-2H_{i}t} \sim e^{-2N}$$

Later radiation and matter-dominated eras can comparatively very slowly drive $\Omega$ away from 1, but not by much.

The horizon problem goes away. The horizon distance also grows exponentially; consult the book’s calculations for $d_{\text{hor}}$ with $a \propto e^{H_{i}t}$ for the details. Watch out for one possible source of confusion. Since the horizon distance indicates the size of the universe which is causally connected at any given time, an exponentially growing $d_{\text{hor}}$ might suggest more and more of an inflating universe becoming causally connected. The key point is just the opposite: an originally causally connected volume of the early universe gets exponentially inflated such that much of it is no longer causally connected after inflation.

Finally, the monopole puzzle also goes away, as inflation results in a colossal dilution of the monopole density, provided they form before inflation.

10.5

Classic inflation relies on a scalar field to provide an effective cosmological constant. Such a field $\phi(\vec{r}, t)$ takes on a scalar value for every point in space and time, as opposed to, say, an electric field, which takes on vector values. When you quantize such a field, you get spin-0 particles associated with the field.

Now, you need some more background in quantum field theory and particle physics to understand some of the technical details, but there is a lot that doesn’t really require such a background. The most basic notion is that is a potential energy density $V(\phi)$ associated with the field. This potential $V(\phi)$ is symmetric with respect to the symmetry transformation $\phi \to -\phi$, but the potential minimum is not located at $\phi = 0$, which is the only value for which $\phi = -\phi$. (See figure 10.4 for a classic potential for inflation.) At high energies or temperatures, the inflaton field respects the symmetry, starting out around $\phi = 0$. This is, however, not the ground state: it’s an unstable (metastable)
equilibrium state of a “false vacuum” (in quantum field theory, “vacuum” often means “ground state”). As things cool down, the field very gradually (notice that the slope of \( V(\phi) \) close to \( \phi = 0 \) and for a long stretch after is very gentle) relaxes toward one of its minima, a “true vacuum.” (Which one? That’s random, as with spontaneous symmetry breaking in general, of which this is a special case.)

Cosmologically, the key equations are the energy density and the associated pressure:

\[
\epsilon_\phi = \frac{1}{2} \frac{1}{\hbar c^3} \dot{\phi}^2 + V(\phi) \quad \text{and} \quad P_\phi = \frac{1}{2} \frac{1}{\hbar c^3} \dot{\phi}^2 - V(\phi)
\]

The energy density equation is easier to understand; it looks very much like a kinetic energy term and a potential energy term. The pressure is more difficult. It may be best to just accept that for now, as one of those technical details for which you lack the background at the moment.

And here’s the thing: if \( \dot{\phi}^2 \ll \hbar c^3 V(\phi) \), we have \( \epsilon_\phi \approx -P_\phi \), which is an equation of state where \( P \approx w \epsilon \) with \( w = -1 \), just like a cosmological constant.

Together with the fluid equation \( \dot{\epsilon}_\phi + 3H(\epsilon_\phi + P_\phi) = 0 \), we can work all this into an equation for the time evolution of the field,

\[
\ddot{\phi} = -3H \dot{\phi} + \hbar c^3 \left( -\frac{dV}{d\phi} \right)
\]

This looks very much like an equation of motion for a particle with location \( x \), velocity \( \dot{x} \), acceleration \( \ddot{x} \),

\[
\ddot{x} = -\frac{D}{m} \dot{x} + \frac{1}{m} \left( -\frac{dV}{dx} \right)
\]

where \( m \) is the mass and \( D \) is a constant associated with drag. On the right side of the equation, we have a nonconservative drag force that is in the opposite direction of the velocity, and a conservative force \( F = -dV/dx \). You know the behavior that results from such a differential equation: movement toward a potential minimum, with damped oscillations around the minimum dying down due to energy loss to the nonconservative drag or friction term. Indeed, \( 3H \dot{\phi} \) is called the “Hubble friction.”

The Hubble friction is also useful to repopulate the universe after the ridiculously high inflation brings the number density of everything, not just
monopoles, down to absurdly low numbers. The friction reheats the universe, producing lots and lots of photons to radiate away the inflaton energy.

Exponential inflation also smooths out the universe, flattening it not just at cosmological scales but locally as well. But at smaller than cosmological scales, the universe is not homogeneous and isotropic; it is decidedly lumpy. Well, ordinary gravity is a purely attractive force, so any local variations in density will lead to instabilities: concentrations of mass growing in a positive feedback loop. These initial variations come from microscopic-scale quantum fluctuations inflated to cosmic dimensions.

Another interesting note is the connection of inflation to theories that postulate multiple universes. After all, with inflation, we have our observed universe as a bubble of true vacuum inflating within a false vacuum. Why should there be just a single bubble? Plus we have multiple true vacuum states. In fact, in many cosmological theories in play today, multiple universe-bubbles of various sorts appear very naturally. This introduces conceptual difficulties associated with finding evidence for multiple universes—we might not even in principle have direct ways of obtaining observational evidence. We then get into ingenious ways to explore possibilities for indirect evidence, but that’s more advanced work.

Recitation and Homework

If you want to do a recitation problem, 10.2 is suitable. For homework, do 10.3; we’ll combine this with chapter 11.

10.3 is long, but it should be straightforward. A lot of it is applying what you’ve learned for universes dominated by \( \Lambda \), radiation (after the false vacuum decay), and equality between matter and radiation afterwards, that you’ve seen in chapter 5. Occasionally you’ll need to pull up numbers or relationships between \( H_0 \) and \( t_0 \) and so forth; use the Benchmark Model for that. When the false vacuum decays, convert the \( \Lambda \) energy density entirely to blackbody radiation, and use the characteristic \( \propto T^4 \) relationship.