More Gravity

1 More black holes

You might have an image of black holes as cosmic vacuum cleaners that suck everything in. It's a bit more complicated, though the gravitational pull of a black hole can be enormous indeed.

We found that a black hole with the mass of the Earth had an event horizon radius of 8.85×10^{-3} m. Let's see what would happen if we were a distance of 1.00 m from the center of such a black hole. Now, we have to be careful. As you move into regions of high gravity, just as in special relativity, space and time intervals are affected. Time stretches (clocks slow down) in high gravity. You wouldn't notice anything slowing down as you approached a black hole, but once you came back to a region with lower gravity, you would find that less time would have passed for you compared to those you left behind.

Near the event horizon of a black hole, space and time begins to warp quite strongly, so Newtonian gravity, $F_G = G\,\frac{mM}{r^2}$, isn't quite accurate anymore. The situation is similar to special relativity when speeds are close to c and Newtonian physics starts to become unreliable. Sill, 1.00 m is a large distance compared to the size of the black hole itself, and so we should expect Newtonian gravity to at least give us an approximate idea of what happens.

The acceleration due to gravity at a distance r to a mass M is

$$a_G = \frac{F_G}{m} = G\,\frac{M}{r^2}$$

To test it, look up the mass of Earth and the radius of the earth, and plug them in to get $a_G = 9.8\,\mathrm{m/s^2}$. Now do the same, but with $r = 1.00\,\mathrm{m}$. You should end up with a large acceleration. This might make you worry that nearby objects have to be sucked into the black hole. Not necessarily. There's a simple way to avoid falling in. Think about it for a minute.

You will have found $a_G = 3.98 \times 10^{14}\,\mathrm{m/s^2} = 0.0013\,c/\mathrm{ns}$. That's ridiculously large. If you were in a spaceship blasting its engine trying to keep you from falling into this black hole, the normal force from the floor of the spaceship would crush you to a puddle.

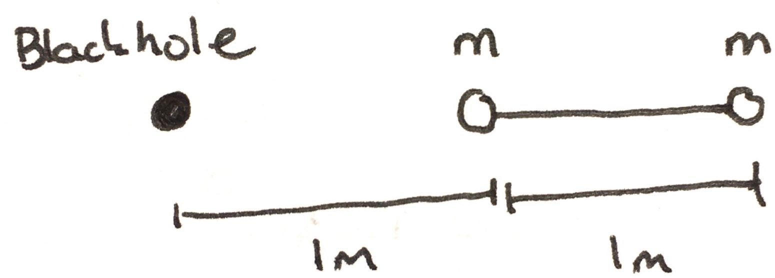
To stop from falling in, though, all you need to do is to be in orbit around the black hole. Just like the Moon is gravitationally attracted to the Earth but does not crash onto the Earth, you would avoid being sucked in. You would have to go very fast, though. Setting the centripetal acceleration $v^2/r = a_G$, we can calculate $v = 2 \times 10^7$ m/s. That's large, but not so near the speed of light, so our approximations should be good enough.

The more serious problem close to a black hole is tidal stretching. When you stand on the surface of the Earth, your head is about 1.5 m farther away from the center of the Earth than your feet, and therefore feels a *very* slightly smaller force of attraction. The difference in between is a tidal stretching force. Say your head and feet were both 2 kg. Then,

$$\begin{aligned} \Delta F &= G\frac{Mm}{r_1^2} - G\frac{Mm}{r_2^2} = GMm\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right) \\ &= (6.67 \times 10^{-11})(5.97 \times 10^{24})(2)\left(\frac{1}{(6.36 \times 10^6)^2} - \frac{1}{(6.36 \times 10^6 + 1.5)^2}\right) \\ &= 9.2 \times 10^{-6}\,\mathrm{N} \end{aligned}$$

You can't notice this. But near a black hole, you'd be torn apart, and being in orbit would not help at all.

Say we have two very compact objects with a mass of $m = 1.00$ kg each, tied to each other by a thin thread. The first object is at $r_1 = 1.00$ m to our black hole, and the second is at $r_2 = 2.00$ m. What would the tension ΔF in the thread be?



The tidal stretching is now an enormous

$$\begin{aligned}\Delta F &= G\frac{Mm}{r_1^2} - G\frac{Mm}{r_2^2} = GMm\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right) \\ &= (6.67\times 10^{-11})(5.97\times 10^{24})(1)\left(\frac{1}{1^2} - \frac{1}{(1+1)^2}\right) \\ &= 5.97\times 10^{14}\,\mathrm{N}\end{aligned}$$

No thread, made of whatever substance, can withstand such a tension force.

In other words, long before you get to the event horizon of an Earth-mass black hole, you'd be stretched into spaghetti, then stretched into a long liquid mass of biological material, and on and on.

By the way, the reason it's called a tidal force is because the tides we see in the Earth's oceans have a similar cause. The Moon's gravity attracts the water close to it with a force stronger than it attracts the waters on the far side of the Earth. The difference causes a slight bulge in the oceans, complicated by the tidal effects of the sun, time delays, and coastal features.

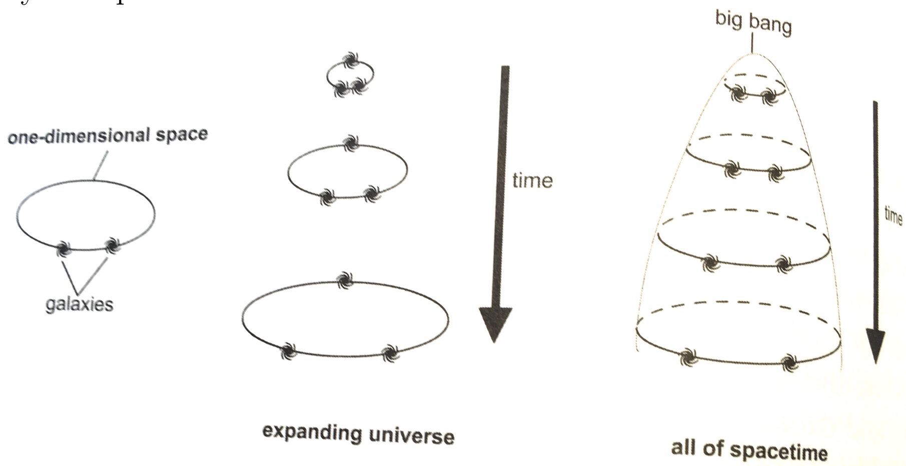
2 The Big Bang

Once we figure out gravity, we can start treating all of space and time—everything, all the past and all the future—as a single geometric object. We can do *cosmology*. And one of the most interesting results we have found, both in our equations and in our astronomical observations, is that the universe is expanding. Space itself is literally stretching out over time!

If we go back far enough, about 14 billion years, all the energy in the universe then must have been compressed into a very small, incredibly hot volume. This is the famous Big Bang theory. You should not, by the way, imagine the universe as coming into being at a certain time, exploding from a point into what had been empty space. *Space itself* is expanding—there is no prior empty space in which an explosion could take place. And we haven't yet invented the physics to help us intelligibly talk about time before the Big Bang. (But we're working on it.)

Now, a curved spacetime is very hard to visualize, especially since I can't make 5-dimensional drawings. But just as a kind of visual metaphor, imagine that our space was 1-dimensional rather than 3. In fact, say it was a circle, with galaxies placed on various points on the circle. Expanding space would just be enlarging the circle, and therefore making the distances between galaxies larger. And if you extrapolated back far enough in time, the circle would shrink down into a point, and then vanish altogether.

In fact, let me steal an illustration from one of the books I wrote, where I try to explain this idea:



Notice that this 1+1-dimensional metaphorical universe is kind of like a half-sphere when you take it as a whole. On our globe, we have latitude and longitude to fix our position. In this universe, time is like latitude, space is like longitude. Going back in time to the Big Bang is like going North: increasing our latitude, until we get to the North Pole at a latitude of 90°. Talking about a time before the Big Bang might seem to make sense; after all, if we talk about 14 billion years ago, why not 15 billion years ago? But what if you tried to go further North on the surface of the earth than the North Pole? It might seem that if you can get from a latitude of 85° North to 90° North, we should be able to go from 90° North to 95° North. But we can't—there is no such place. There is no time before the Big Bang any more than there is a point on the globe North of the North Pole.

That is the case with the standard Big Bang model. We know this model must break down as we approach the time of the Big Bang itself. Among other reasons, we start violating quantum mechanics, and physicists don't think that should happen. But we have not been able to devise a good theory of quantum gravity yet, and depending on whatever your favorite cosmological model is (all are still half-baked) it may or may not make sense to talk about times earlier than the Big Bang. We just don't know.