Optics & Waves

39  Note that $V(t)$ is an odd function, so only the $\sin(n \omega t)$ components should exist. The Fourier components will be

$$b_n \propto \int_0^{2\pi/\omega} dt \sin(n \omega t) V(t)$$

You will notice at a glance that that integral is zero for $n = 2$ (indeed, all even values of $n$). So only (B) survives.

50  The pipe length changes very little with temperature, so $\lambda$ is constant. If $v = \lambda f$, and $v$ is 3% lower, then so must be $f$.

51  Of the electric field on an electromagnetic wave, a linear polarizing filter lets through only the component along its axis of orientation. Therefore, if there is an angle $\theta$ in between, the electric field that makes it through picks up a $\cos \theta$ factor; the amplitude $E \rightarrow E \cos \theta$. The intensity goes like the square of the field amplitude, so the outgoing intensity $I \rightarrow I \cos^2 \theta$. Unpolarized light has a random mixture of $E$ orientations, so averaging out results in polarized light with $I \rightarrow \frac{1}{2}I$ making it through.

In this problem, the intensity picks up a factors of a half after each filter: $\frac{1}{2} \cos^2 45^\circ \cos^2 (90^\circ - 45^\circ) = \frac{1}{8}$.

79  The group velocity $v_g = \frac{\partial \omega}{\partial k}$, and the phase velocity $v_p = \frac{\omega}{k}$. For $k_1 < k < k_2$, you can see these are the opposite signs, even if you forget which definition corresponds to which velocity.

67  Notice that the intensity varies with angle over a half-turn of the filter ($\cos 2\theta$), indicating some linearly polarized incident light (see number 51 above), but also that $A > B > 0$, and therefore the intensity is never zero, indicating some unpolarized light.

68  A useful optics formula you should remember for such exams is that the angular resolution in radians $\sim 1.22 \frac{\lambda}{D}$, where $D$ is the characteristic diameter or size of your optical instrument. Here, $1.22(5500 \times 10^{-10} \text{ m})/D = 8 \times 10^{-6}$, so $D \approx 0.1 \text{ m}$. 

Phys 345 GRE segment comments
(A) and (C) have improper units. At the limit when $\theta = \theta' = 0$, there should be no angular spread: $\delta \theta' = 0$. So the answer must be (E).

The easiest way to do this is with some ray-tracing on the diagram: draw a ray that goes through the focal point (this will be bent parallel to the axis) and a ray that goes parallel to the axis (this will be bent toward the focal point). The image will form where the rays intersect. In this case, you will see that the image forms to the right of the second lens, at a distance less than its focal length.

You have to know what you’re doing, but if you do, it’s simple: the magnification $10 = f_o/f_e$, and the path length is $f_o + f_e = 1 + 1/10 = 1.1$ m.

The object is closer than the focal point, so the image will be virtual and behind. You can see this by ray-tracing: the ray parallel to the axis and the ray through the focal point will diverge, so you have to trace back to the point where the rays look like they are coming from, which is behind the mirror.

With a grating or a double-slit, $\sin \theta = m\lambda/d$. Here, the distance between neighboring slits, $d = 1/2 \times 10^5 = 5 \times 10^{-6}$ m. You get $\sin \theta = \lambda/d \approx 0.1$. Make a small-angle approximation, so $\theta \approx \sin \theta \approx 0.1$ rad. With $\pi \approx 3$, the angle is $\theta \approx 0.1(180^\circ)/\pi \approx 0.6^\circ$.

You just have to remember the Bragg diffraction equation, $2d \sin \theta = n\lambda$. For the largest possible $\lambda$, $\sin \theta = 1$ and $n = 1$.

(I) is nonsense. (II) is plausible; don’t eliminate it. (III) and (IV) are also plausible, since you might remember that there is such a phase shift when reflecting from an interface where the light is coming from a medium with a lower index of refraction and going into a higher index medium, but no phase shift the other way around. And indeed, if you add (II), (III), and (IV) together, you get two reflected waves that are nearly completely ($\pi$) out of phase, canceling each other out.

The light has a seriously large redshift, which indicates a speed quite close to the speed of light away from the Earth, which is (D).
Without the gas, we would have $5 \times 10^{-2}/500 \times 10^{-9} = 10^5$ wavelengths in the cell. Fringes are separated by a half-wavelength, so 40 fringes means 20 wavelengths. This means we have a shift of $20/10^5 = 2 \times 10^{-4}$ that the gas accounts for, with $n = 1 + 2 \times 10^{-4}$.

Again, $\sin \theta = m\lambda/d = m(c/f)/d$. For small angles, doubling $f$ will halve the fringe separation.

Again, the angular resolution $\theta \approx 1.22 \lambda/D$, resulting in a diameter of $D \approx 1.22(600 \times 10^{-9})/(3 \times 10^{-5}) = 2.44 \times 10^{-2}$ m.

The wavelength of the green light has to be $85865/100000 \approx 0.86$ times the red wavelength: $0.86(630) \approx 630 - 60 - 30 = 540$ nm.

Skip this one. But if you attempt it, $w \sin \theta = n\lambda$ is single-slit diffraction, while $d \sin \theta = m\lambda$ is for a double-slit situation. So for a missing maximum, $m/w = n/d$, for $m, n$ integers. It must be that $d > w$, and therefore $m < n$. (D) is the one that fits these conditions.

Both interfaces are from a small index of refraction into a large one, so we can ignore the effect of reflective phase shifts. Since we want a phase shift of $n2\pi$, or a path length difference that is an integer multiple of a wavelength, look for a path length difference of $\lambda$, which means a layer thickness of $\lambda/2$. You also need to account for shortening of $\lambda$ due to $n = 1.2$, which means $\lambda \rightarrow \lambda/1.2$. The thickness, then, is $(480/2)/1.2 = 200$ nm.