Classical physics breaks down

The physics you’ve learned up to now is often called “classical physics”—it’s a physics where we can easily imagine particles zipping around and interacting under the influence of forces. It’s an impressive intellectual structure, but as with everything we’ve encountered so far in physics, it works well in a particular range of circumstances but breaks down when you push it to extremes. In this case, when you start looking at very small objects—molecules, atoms, subatomic particles—classical physics breaks down and we need to invent a new description, quantum physics, to understand what’s going on.

Let’s look more closely at the simplest of atoms, Hydrogen. If I were to ask you to draw a picture of H, you would probably come up with the planetary model of an atom. You have a proton and an electron, which interact by an electrical attraction, which is an inverse-square force just like the gravity keeping the Earth bound to the Sun. The proton is 2000 times more massive than the electron, so the center of mass of the system is very close to the proton. This means that the proton is almost stationary and the electron can be treated as revolving around the proton. This is just like the Sun is much more massive than the Earth, and we treat the Earth as revolving around the Sun. So we draw a circular orbit with radius $r$ for the electron ($e^-$) around the proton ($p$), and indicate the electrical attraction $F_E = ke^2/r^2$, which provides the centripetal force $m_e v^2/r$ for circular motion. ($e$ is the electron charge and $m_e$ is the electron mass.)

Go ahead and draw such a picture of H, with the $e^-$ in orbit around the $p$, indicating the forces, the radius, and the velocity of the $e^-$. 
Because we’ll need it soon, let’s also figure out the total energy of the electron in orbit. The potential energy due to the interaction is the usual charge times voltage, \( U_E = eV = -\frac{ke^2}{r} \), using the voltage due to a point particle and the fact that the \( e^- \) and \( p \) have charges with opposite signs. The kinetic energy is \( K = \frac{1}{2}m_e v^2 = \frac{1}{2}r F_E = \frac{1}{2} \frac{ke^2}{r} \). Therefore,

\[
E = \frac{1}{2} m_e v^2 - k \frac{e^2}{r} = -k \frac{e^2}{2r}
\]

The negative energy is normal—H is a bound state of the \( e^- \) and \( p \), so the energy is lower than the \( E = 0 \) we’d have when they would be separated at \( r \to \infty \).

There is actually something wrong with this picture. As the \( e^- \) circles around the \( p \), its acceleration \( a \neq 0 \). And accelerating charges radiate electromagnetic waves. That is, in fact, how you get broadcast radio and TV: an antenna is just a circuit, where charges are circulating, therefore accelerating, and therefore radiating. (For visible light, you just jiggle charges such as electrons at higher frequencies.) But electromagnetic waves, like any wave, transmit energy. Energy is conserved, and therefore the energy that is broadcast into space must come from somewhere. That somewhere can only be the electron in orbit: its energy must decrease. We just calculated \( E \propto -1/r \). If \( E \) is to decrease (become even more negative), \( r \) must get smaller. Therefore, as the electron revolves, its radius cannot stay constant. The radius must be getting smaller all the time!

Draw a picture, then, of what you think the trajectory of the \( e^- \) must look like when you account for the energy continually being radiated away.
The $e^-$ will spiral down toward the $p$. The electric interaction is very strong, so the rate of energy loss is quite high—the atom will collapse in a fraction of a second. According to classical physics, atoms cannot exist.

There are many experiments that classical physics cannot account for; indeed, classical physics can't explain much when it comes to the behavior of very small particles. Therefore we need quantum physics.

2 Particles or waves?

We’ve just mentioned electromagnetic waves. Recently we’ve also encountered photons, which are particles of light. One basic property of photons is that their energy is proportional to their frequency:

$$E = hf$$

where $h$ is a proportionality constant, “Planck’s constant.”

This property of photons is important in chemistry. Typical ionization energies and molecular bond energies are around 1 to 10 eV. This means frequencies close to $f = E/h \sim 10^{15}$ Hz, which is visible to ultraviolet (UV) light. Indeed, visible and UV exposure can give you a sun tan as photons with such frequencies have enough energy to break chemical bonds when absorbed. DNA in skin cells, for example, can be damaged, leading to skin cancer. But lower frequency light, such as infrared (IR) from a heat lamp, can’t give you skin cancer no matter how intense it is—it can only cook you. Electromagnetic radiation with much higher frequencies than visible light, such as X-rays with photon energies around $\sim 10^4$ eV, are energetic enough to knock out tightly bound inner-shell electrons of atoms, and can cause chemical havoc. They’re real cancer hazards, depending on exposure.

You’re probably vaguely familiar with all this, from your experience with chemistry. But look more carefully at the equation $E = hf$. The left side
of the equation $E$, is the energy of a particle. A particle is an all-or-nothing affair. It’s localized. The photon is absorbed and it breaks the chemical bond, or it misses and it goes by. But the right side of the equation has $f$, the frequency. Frequency is about waves; it doesn’t apply to particles. It implies something changing, waving up and down. What could that possibly be? We learned about waves and particles as two distinct ways of transmitting information—how can something like a photon do both? When you think about it, you should have a hard time making sense out of $E = hf$.

Experimentally, we can measure both particle and wave properties of photons. We can, for example, dim a light source so much that it only emits a photon every second or less. And we can detect those photons hitting a screen—we detect blips, with energy deposited on a particular spot, which is exactly particle behavior. And yet, we know that if we pass light through a double-slit setup or a diffraction grating, we see the maxima and minima of brightness that are signatures of interference. Only waves can do interference.

There’s actually no inconsistency or mysterious duality here. When you shoot photons at a screen, you will always observe blips—particles. But it turns out that if you shoot single photons through a double-slit setup, the location in which the photon hits the screen is unpredictable. It’s random. But a photon is more likely to hit the screen in one of the maxima, much less likely around a minimum. When you send lots and lots of photons through a double slit setup, over time, on average, there will be a lot more blips around the maxima, a lot fewer among the minima. Here’s an image which I stole from online that illustrates the gradual buildup of the interference pattern from random individual photon blips:
In quantum mechanics, particles are still particles, but their behavior is random—described by probability distributions rather than exact, deterministic equations. When we have very large numbers of particles, the statistical law of large numbers comes into play: the average values become very predictable, and those are what the equations of classical physics describe. The waviness comes about because the probability distributions are determined by a “wave function,” which behaves according to a wave equation.

### 3 Wave functions

The quantum state of a particle who lives on the $x$-axis is described by a wave function $\psi(x)$. Technically, $\psi$ is a complex function, with a real part and an imaginary part, but since we’re just trying to understand the basics, let’s just take $\psi$ to be an ordinary function, like a sine wave. Remember, from our study of waves, that we started out investigating pure sine waves because they were mathematically simple, and had a well-defined wavelength and frequency. Then, we observed that more complicated waveforms—any function, really—could be built out of basic sine waves that we added together. (Typically this is an infinite sum of waves with different wavelengths and amplitudes.)

In quantum mechanics, wavelength is associated with the particle momentum, $p$. For photons, in relativity, $E = pc$, and if we combine that with the quantum $E = hf$, we get $p = hf/c = h/\lambda$. In quantum mechanics, this is true for every particle, not just photons. Again,

$$p = \frac{h}{\lambda}$$

Just as with $E = hf$, this equates a particle property with a wave property. But now we know what the wave is: the wave function $\psi$, and if $\psi$ has a definite $\lambda$ value, it must be a pure sine wave.

Now, draw a pure sine wave, with $\psi$ on the vertical and $x$ on the horizontal axis. Remember that the intensity of a wave is proportional to its amplitude squared, $I \propto A^2$. When you’re dealing with a single particle on the $x$-axis, the relevant concept of intensity is the probability of finding the particle at the location $x$. Call this $P(x)$. Therefore $P(x) \propto A^2$. As the amplitude gets larger, so should the probability. Ask yourself: with a pure sine wave, how does the amplitude, and therefore the probability, change as a function of $x$? Sketch a graph of $P(x)$ vs $x$.
The amplitude of a sine wave never changes. Therefore, the probability should be constant. Notice that everywhere along the $x$-axis is equally probable. We know nothing about the particle location; if we did, we could distinguish one location from another. Equivalently, we can say that our uncertainty about the location is as large as it can be, which is infinite. Let’s call our $x$-uncertainty $\Delta x$; therefore $\Delta x = \infty$.

In your probability and statistics coursework, you learned about averages and standard deviations. A standard deviation is a measure of the width of a distribution; the wider it is, the more our uncertainty. $\Delta x$ is the standard deviation of $P(x)$. As you can see, a flat probability is an infinitely wide probability distribution.

A particle described by a wave function that is a pure sine wave has a definite momentum $p = h/\lambda$. So we know $p$ exactly; our uncertainty $\Delta p = 0$. The probability distribution for momentum, $P(p)$, has zero width; it’s a narrow spike. But our certainty about $p$ comes at a price of complete uncertainty about $x$; $\Delta x = \infty$.

Now imagine a case where the wave function allows us to know something about $x$—the amplitude varies.

Draw the $P(x)$ vs $x$ graph for this $\psi(x)$. 
The probability should look like

This \( \psi \) is not now a pure wave; we would build \( \psi \) by adding up lots of waves of different wavelengths. In other words, we gain some knowledge of \( x \), so that \( \Delta x \) is no longer infinite. But the price we pay is that we are no longer certain that the momentum is \( p = \hbar/\lambda \), because we no longer have a single \( \lambda \). We have introduced uncertainty into the momentum, so that \( \Delta p \) is no longer zero. \( P(p) \) will now look like

In short, there is a tradeoff between knowledge about position and knowledge about momentum. This is expressed by the famous uncertainty principle:

\[
\Delta x \Delta p \geq \frac{\hbar}{4\pi}
\]

A lot of people have talked a lot of nonsense about quantum physics, including the uncertainty principle. It’s not deeply mysterious—it isn’t a metaphysical dictum about the limits of knowability. The uncertainty principle directly follows from the fact that quantum systems are described by wave functions. It’s all about waves.

Here’s a quick calculation. Let’s say that we know a particle with mass \( m \) is confined to between \( x = 0 \) and \( x = L \). (It’s in a one-dimensional box.) What would then be a good estimate for \( \Delta x \)? What, as a result, would be a good estimate for \( \Delta p \)?
If the particle has $0 \leq x \leq L$, the width of the probability distribution $P(x)$ has to be around $L$. (What else?) That’s a good estimate for the position uncertainty: $\Delta x \approx L$. Given that, the minimum uncertainty for momentum is $\Delta p \approx h/(4\pi \Delta x) = h/(4\pi L)$.

A particle confined to a box has kinetic energy $K = p^2/2m$. This is positive, regardless of the sign of $p$. Indeed, the only way you can get $K = 0$ is if $p = 0$ and we were exactly certain that $p = 0$: if $\Delta p = 0$. Since we have established that $\Delta p > 0$, this means that the energy can’t be zero! We can quickly estimate the minimum energy. For a distribution $P(p)$ centered around an average $p = 0$, the average $(p^2)_{\text{avg}} \approx (\Delta p)^2$. So the minimum energy is

$$K_{\text{min}} \approx \frac{(p^2)_{\text{avg}}}{2m} \approx \frac{(\Delta p)^2}{2m} = \frac{h^2}{32\pi^2 m L^2}$$

This is a rough estimate; the real quantum calculation gives a factor of $\frac{1}{8}$ rather than $\frac{1}{32}$. But still, getting the right functional dependence on $m$ and $L$, and being off by only a factor of a quarter isn’t at all bad.

Moreover, we have an important qualitative result here. A classical particle has a definite position and a definite momentum, and when confined, can have any energy, including zero. None of these things are true for quantum (and real world!) particles. In fact, bound states of quantum particles can’t just have any energy: their energies can only be within a set of discrete energy levels. (The energies are said to be “quantized.”)

This, finally, is how atoms can exist. Hydrogen has a minimum energy state; the $e^-$ cannot lower its energy indefinitely by spiraling down toward the $p$. In fact, the $e^-$ does not rotate around the $p$ at all. In its lowest energy state, the $e^-$ has zero angular momentum, and a probability distribution centered on top of the proton. Here’s a standard textbook illustration of the spatial probability distribution I stole, where the density of dots indicates the probability density:
The important point is that the *uncertainty* in position is not zero! So the energy is higher than the classical minimum (which is $-\infty$). And there is no electromagnetic radiation emitted, because the $e^-$ is *not rotating*.

That’s all I will tell you about basic quantum mechanics; beyond this, it rapidly starts getting technical and very mathematical. But you should have now learned enough to know a few things.

First, you should start moving beyond the way pop culture tends to identify the word quantum with *magic*. Quantum physics is weird in the sense that it goes against our intuitions, but it’s just physics. It’s waves. If anyone tries to tell you about “quantum healing” or imply that quantum mechanics underlies a device they’re selling that will do wonderful things for you, they’re almost certainly mistaken or worse. Do a search on “Quantum University” and see what sort of nonsense is out there. Don’t fall for it.

Second, you should again realize that the world does not work according to our common sense. Just as in relativity, our intuitive feel for how things should behave is just not very useful.