Quantum

98 Looking at the matrix, it’s not equal to its own transpose complex-conjugated. In other words, it’s not Hermitian. Only Hermitian matrices have all real eigenvalues. So (B) is wrong.

89 The odd-parity \( \psi(-x) = -\psi(x) \) eigenstates of the full simple harmonic oscillator will also be eigenstates of this potential, since they will have \( \psi(0) = -\psi(0) = 0 \). Odd-parity SHO eigenstates are the ones with odd \( n \). So (E) is correct.

81 \( L^2 \) eigenvalues are \( l(l+1)\hbar^2 \) with \( l = 0, 1, 2, \ldots \). \( L_z \) eigenvalues are \( m\hbar \) with \( m = -l, -l+1, \ldots, l-1, l \). Since \( l(l+1) = 6, l = 2, \) and \( m = -1 \). Therefore (B).

30 (I) is fine, with its exponential decay. (II) does not decay and just oscillates; it cannot be a bound state. (III) decays, but it blows up at \( r = 0 \), which can’t happen.

99 First order perturbative corrections to the energy levels, with a perturbation Hamiltonian \( H' \), are the expectation values \( \langle H' \rangle = \langle n | H' | n \rangle \). A static electric field will have a voltage \( \sim E_z \), and the perturbation energy will be \( \sim eE_z \). The ground state of \( H \) is spherically symmetric, so \( \langle z \rangle = 0 \), and therefore there is a zero first order correction to the energy.

43 You have to work out the commutator:
\[
[L_x L_y, L_z] = L_x L_y L_z - L_z L_x L_y = (L_x L_z + i\hbar L_y)L_y \\
= L_x L_y L_z - i\hbar L_y^2 - L_z(L_y L_z - i\hbar L_x) = i\hbar L_x^2 - i\hbar L_y^2
\]
There don’t seem to be any shortcuts.

29 (A) does not go to 0 as \( x \to \pm \infty \). (C) has a discontinuity. (D) has a discontinuity in \( \partial \psi / \partial x \) at a finite discontinuity in \( V \). (E) is oscillating in the classically forbidden region. (B) is fine.

44 The given state only has \( n = 1, 2, 3 \) components, which means only energies of \( 1^2 E_1, 2^2 E_1, 3^2 E_1 \) have non-zero probability of being observed.
If you have experience with H, you know the answer is (C). Or you can note that the radial probability \( P(r)dr = |\psi_0|^2 4\pi r^2 dr \), and find the maximum by \( dP(r)/dr = 0 \), which is a slog.

Spin singlets and triplets refer to the one spin combination that is anti-symmetric under particle exchange, \( \frac{1}{\sqrt{2}}(|\alpha\beta\rangle - |\beta\alpha\rangle) \), and the three symmetric combinations \( |\alpha\alpha\rangle \), \( |\beta\beta\rangle \), and \( \frac{1}{\sqrt{2}}(|\alpha\beta\rangle + |\beta\alpha\rangle) \). We’re looking for a triplet, so (I) and (III) work.

\[ \langle p \rangle = \int dx \psi^*_n (-i\hbar \partial/\partial x) \psi_n = 0. \]
Alternatively, \( \sin kx = (e^{ikx} - e^{-ikx})/2i \), which is an equal-weight \( \langle |i| = | - i| \rangle \) combination of momenta to the right and to the left, so the expectation value is zero.

This is a statement of orthonormality.

(A) has energies bounded from above. (C), (D), and (E) have only one possible energy level. (B) is fine; it just tells you that the measured energy is always greater than or equal to the ground state energy.

The first order shift is
\[
\langle 2|V(a + a^\dagger)^2|2 \rangle = V\langle 2|(a^2 + aa^\dagger + a^\dagger a + a^4)|2 \rangle = V \left( \sqrt{2}\sqrt{1}\langle 2|0 \rangle + 2\langle 2|2 \rangle + 3\langle 2|3 \rangle + \sqrt{3}\sqrt{4}\langle 4|2 \rangle \right) = 5V
\]

Note the orthonormality of states: \( \langle m|n \rangle = \delta_{mn} \).

The probability of an energy is the magnitude squared of the vector component with that energy. So \( \langle E \rangle = \langle \psi|H|\psi \rangle = \frac{1}{14^2} + \frac{3}{14^2} + \frac{5}{14^2} + \frac{9}{14^2} + \frac{7}{14^2} = \frac{43}{14} \), in \( \hbar\omega \) units.

Always real, which is why physical observables have Hermitian operators.

\( \langle \psi_1|\psi_2 \rangle = 5 + 15 + 2x = 0 \), therefore \( x = -10 \).

This is a plane wave, which is a momentum eigenstate, with \( p = \hbar k \). Check: \(-i\hbar \partial \psi/\partial x = \hbar k \psi \).
28 There are two $m = 3$ components. (Look at the superscripts of the $Y_m^l$ functions—these are spherical harmonics, which are angular momentum eigenfunctions.) Adding up the probabilities gives $\frac{1}{30}(|5|^2 + |1|^2) = \frac{13}{15}$.

27 The momentum uncertainty $\Delta p = \hbar \Delta k$. The uncertainty principle should lead you to expect $\Delta x \Delta p \sim \hbar$, and so $\Delta k \sim 1/\Delta x_0$.

50 This is a statement of the commutator theorem.

56 SHO energy levels are $E_n = \hbar \omega (n + \frac{1}{2}) = \hbar \nu (n + \frac{1}{2})$ with $n = 0$ the ground state.

29 The probabilities of the values of $O$ are the magnitude squared of the coefficient of the basis vectors which are eigenvectors of $\hat{O}$. So the expectation value is $\frac{1}{6}(-1) + \frac{1}{2}(1) + \frac{1}{3}(2) = 1$.

46 The free particle energy is purely kinetic, and the momentum is $p = \hbar k$. Therefore $E \propto k^2$. When it steps up, it will have $E - V \propto k^2$. Since $\lambda \propto 1/k$, the new $\lambda \propto (E - V)^{-1/2}$.

83 The column vector representation of (A) is $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Multiplying the $\sigma_x$ matrix with that, we get $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, which is not $-1$ times itself. Checking the options one by one, we find (C) works.