30. Which of the following functions could represent the radial wave function for an electron in an atom? (r is the distance of the electron from the nucleus; A and b are constants.)

I. \( A e^{-br} \)

II. \( A \sin(br) \)

III. \( A/r \)

A) I only
B) II only
C) I and II only
D) I and III only
E) I, II, and III

44%

99. In perturbation theory, what is the first order correction to the energy of a hydrogen atom (Bohr radius \( a_0 \)) in its ground state due to the presence of a static electric field \( E \)?

A) Zero
B) \( eEA_0 \)
C) \( 3eEA_0 \)
D) \( \frac{8e^2EA_0^3}{3} \)
E) \( \frac{8e^2E^2a_0^3}{3} \)

21%

81. Which of the following is the orbital angular momentum eigenfunction \( Y_1^m(\theta, \phi) \) in a state for which the operators \( L^2 \) and \( L_z \) have eigenvalues \( 6\hbar^2 \) and \( -\hbar \), respectively?

A) \( Y_1^1(\theta, \phi) \)
B) \( Y_2^{-1}(\theta, \phi) \)
C) \( \frac{1}{\sqrt{2}} [Y_1^1(\theta, \phi) + Y_2^{-1}(\theta, \phi)] \)
D) \( Y_1^2(\theta, \phi) \)
E) \( Y_1^{-1}(\theta, \phi) \)

50%
29. An attractive, one-dimensional square well has depth $V_0$ as shown above. Which of the following best shows a possible wave function for a bound state?

(A) 

(B) 

(C) 

(D) 

(E) 

44. The energy eigenstates for a particle of mass $m$ in a box of length $L$ have wave functions $\phi_n(x) = \sqrt{\frac{2}{L}} \sin(n\pi x / L)$ and energies $E_n = n^2\pi^2\hbar^2 / 2mL^2$, where $n = 1, 2, 3, \ldots$. At time $t = 0$, the particle is in a state described as follows.

$$\Psi(t = 0) = \frac{1}{\sqrt{14}} [\phi_1 + 2\phi_2 + 3\phi_3]$$

Which of the following is a possible result of a measurement of energy for the state $\Psi$?

(A) $2E_1$
(B) $5E_1$
(C) $7E_1$
(D) $9E_1$
(E) $14E_1$

93. The solution to the Schrödinger equation for the ground state of hydrogen is

$$\psi_0 = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0},$$

where $a_0$ is the Bohr radius and $r$ is the distance from the origin. Which of the following is the most probable value for $r$?

(A) 0
(B) $a_0/2$
(C) $a_0$
(D) $2a_0$
(E) $\infty$

82. Let $|\alpha\rangle$ represent the state of an electron with spin up and $|\beta\rangle$ the state of an electron with spin down. Valid spin eigenfunctions for a triplet state ($^3S$) of a two-electron atom include which of the following?

I. $|\alpha\rangle_1 |\alpha\rangle_2$

II. $\frac{1}{\sqrt{2}} (|\alpha\rangle_1 |\beta\rangle_2 - |\alpha\rangle_2 |\beta\rangle_1 )$

III. $\frac{1}{\sqrt{2}} (|\alpha\rangle_1 |\beta\rangle_2 + |\alpha\rangle_2 |\beta\rangle_1 )$

(A) I only
(B) II only
(C) III only
(D) I and III
(E) II and III
Questions 51-53

A particle of mass \( m \) is confined to an infinitely deep square-well potential:
\[
V(x) = \begin{cases} \infty, & x \leq 0, \\ 0, & 0 < x < a. \end{cases}
\]
The normalized eigenfunctions, labeled by the quantum number \( n \), are \( \psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a} \).

51. For any state \( n \), the expectation value of the momentum of the particle is
   \( \frac{\hbar n\pi}{a} \)
   \( \frac{\hbar n\pi}{a} \cos n\pi - 1 \)
   \( \frac{\hbar n\pi}{a} \cos n\pi - 1 \)
   \( \frac{\hbar n\pi}{a} \cos n\pi - 1 \)
   \[ \text{(A) 0} \]
   \[ \text{(B) } \frac{\hbar n\pi}{a} \]
   \[ \text{(C) } \frac{\hbar n\pi}{a} \cos n\pi - 1 \]
   \[ \text{(D) } -\frac{i\hbar n\pi}{a} \cos n\pi - 1 \]
   \[ \text{(E) } 5.5 \]
   \( \text{41%} \)

52. The eigenfunctions satisfy the condition
\[
\int_0^a \psi_n^*(x) \psi_{n'}(x) \, dx = \delta_{n,n'}, \delta_{n,0} = 1 \text{ if } n = 0, \text{ otherwise } \delta_{n,0} = 0.
\]
This is a statement that the eigenfunctions are
   \( \text{(A) solutions to the Schrödinger equation} \]
   \( \text{(B) orthonormal} \]
   \( \text{(C) bounded} \]
   \( \text{(D) linearly dependent} \]
   \( \text{(E) symmetric} \)
   \[ \text{(A) solutions to the Schrödinger equation} \]
   \[ \text{(B) orthonormal} \]
   \[ \text{(C) bounded} \]
   \[ \text{(D) linearly dependent} \]
   \[ \text{(E) symmetric} \)
   \[ 86% \]

53. A measurement of energy \( E \) will always satisfy
which of the following relationships?
   \( E \leq \frac{\pi^2 \hbar^2}{8ma^2} \)
   \( E \geq \frac{\pi^2 \hbar^2}{2ma^2} \)
   \( E = \frac{\pi^2 \hbar^2}{8ma^2} \)
   \( E = \frac{\pi^2 \hbar^2}{8ma^2} \)
   \( E = \frac{\pi^2 \hbar^2}{2ma^2} \)
   \[ \text{(A) } E \leq \frac{\pi^2 \hbar^2}{8ma^2} \]
   \[ \text{(B) } E \geq \frac{\pi^2 \hbar^2}{2ma^2} \]
   \[ \text{(C) } E = \frac{\pi^2 \hbar^2}{8ma^2} \]
   \[ \text{(D) } E = \frac{\pi^2 \hbar^2}{8ma^2} \]
   \[ \text{(E) } E = \frac{\pi^2 \hbar^2}{2ma^2} \]
   \[ 4.3% \]

94. The raising and lowering operators for the quantum harmonic oscillator satisfy
\[
a^\dagger |n\rangle = \sqrt{n+1} |n+1\rangle, \quad a|n\rangle = \sqrt{n} |n-1\rangle
\]
for energy eigenstates \( |n\rangle \) with energy \( E_n \).
Which of the following gives the first-order shift in the \( n = 2 \) energy level due to the perturbation
\[ \Delta H = V(a + a^\dagger)^2, \]
where \( V \) is a constant?
   \( \text{(A) 0} \]
   \( \text{(B) } V \]
   \( \text{(C) } \sqrt{2}V \]
   \( \text{(D) } 2\sqrt{2}V \]
   \( \text{(E) } 5V \)
   \[ 29% \]

45. Let \( |n\rangle \) represent the normalized \( n \)th energy eigenstate of the one-dimensional harmonic oscillator, \( H |n\rangle = \hbar \omega \left( n + \frac{1}{2} \right) |n\rangle \).
If \( |\psi\rangle \) is a normalized ensemble state that can be expanded as a linear combination
\[ |\psi\rangle = \frac{1}{\sqrt{14}} |1\rangle - \frac{2}{\sqrt{14}} |2\rangle + \frac{3}{\sqrt{14}} |3\rangle \]
of the eigenstates, what is the expectation value of the energy operator in this ensemble state?
   \( \text{(A) } \frac{102}{14} \hbar \omega \]
   \( \text{(B) } \frac{43}{14} \hbar \omega \]
   \( \text{(C) } \frac{23}{14} \hbar \omega \]
   \( \text{(D) } \frac{17}{14} \hbar \omega \]
   \( \text{(E) } \frac{7}{14} \hbar \omega \)
   \[ \text{50%} \]

27. The eigenvalues of a Hermitian operator are always
   \( \text{(A) real} \]
   \( \text{(B) imaginary} \]
   \( \text{(C) degenerate} \]
   \( \text{(D) linear} \]
   \( \text{(E) positive} \)
   \[ \text{82%} \]
QUANTUM

\[ |\psi_1\rangle = 5|1\rangle - 3|2\rangle + 2|3\rangle \]
\[ |\psi_2\rangle = |1\rangle - 5|2\rangle + x|3\rangle \]

28. The states \( |1\rangle, |2\rangle, \) and \( |3\rangle \) are orthonormal.

For what value of \( x \) are the states \( |\psi_1\rangle \) and \( |\psi_2\rangle \) given above orthogonal?

(A) 10
(B) 5
(C) 0
(D) -5
(E) -10

1. The wave function of a particle is \( e^{i(kx-\omega t)} \), where \( x \) is distance, \( t \) is time, and \( k \) and \( \omega \) are positive real numbers. The \( x \)-component of the momentum of the particle is

(A) 0
(B) \( h\omega \)
(C) \( hk \)
(D) \( \frac{h\omega}{c} \)
(E) \( \frac{hk}{\omega} \)

27. If a freely moving electron is localized in space to within \( \Delta x_0 \) of \( x_0 \), its wave function can be described by a wave packet \( \psi(x, t) = \int_{-\infty}^{\infty} e^{i(kx-\omega t)} f(k) \, dk \), where \( f(k) \) is peaked around a central value \( k_0 \). Which of the following is most nearly the width of the peak in \( k \)?

(A) \( \Delta k = \frac{1}{x_0} \)
(B) \( \Delta k = \frac{1}{\Delta x_0} \)
(C) \( \Delta k = \frac{\Delta x_0}{x_0^2} \)
(D) \( \Delta k = \left( \frac{\Delta x_0}{x_0} \right) k_0 \)
(E) \( \Delta k = \frac{1}{k_0^2 + \left( \frac{1}{x_0} \right)^2} \)

50. The state of a quantum mechanical system is described by a wave function \( \psi \). Consider two physical observables that have discrete eigenvalues: observable \( A \) with eigenvalues \( \{a_i\} \), and observable \( B \) with eigenvalues \( \{b_j\} \). Under what circumstances can all wave functions be expanded in a set of basis states, each of which is a simultaneous eigenfunction of both \( A \) and \( B \)?

(A) Only if the values \( \{a_i\} \) and \( \{b_j\} \) are nondegenerate
(B) Only if \( A \) and \( B \) commute
(C) Only if \( A \) commutes with the Hamiltonian of the system
(D) Only if \( B \) commutes with the Hamiltonian of the system
(E) Under all circumstances

56. If \( v \) is frequency and \( h \) is Planck's constant, the ground state energy of a one-dimensional quantum mechanical harmonic oscillator is

(A) 0
(B) \( \frac{1}{3} hv \)
(C) \( \frac{1}{2} hv \)
(D) \( hv \)
(E) \( \frac{3}{2} hv \)
29. The state \( \psi = \frac{1}{\sqrt{6}} \psi_{-1} + \frac{1}{\sqrt{2}} \psi_{1} + \frac{1}{\sqrt{3}} \psi_{2} \) is a linear combination of three orthonormal eigenstates of the operator \( \hat{O} \) corresponding to eigenvalues \(-1, 1, \) and \(2\). What is the expectation value of \( \hat{O} \) for this state?

(A) \(\frac{2}{3}\)

(B) \(\sqrt[6]{7}\)

(C) \(\frac{4}{3}\)

(D) \(\frac{\sqrt{3} + 2\sqrt{2} - 1}{\sqrt{6}}\)

(E) \(\frac{\sqrt{3} + 2\sqrt{2} - 1}{\sqrt{6}}\)

46. A free particle with initial kinetic energy \(E\) and de Broglie wavelength \(\lambda\) enters a region in which it has potential energy \(V\). What is the particle’s new de Broglie wavelength?

(A) \(\lambda (1 + \frac{E}{V})\)

(B) \(\lambda (1 - \frac{V}{E})\)

(C) \(\lambda (1 - \frac{E}{V})^{-1}\)

(D) \(\lambda (1 + \frac{V}{E})^{1/2}\)

(E) \(\lambda (1 - \frac{V}{E})^{-1/2}\)

83. The state of a spin-\(\frac{1}{2}\) particle can be represented using the eigenstates \(|\uparrow\rangle\) and \(|\downarrow\rangle\) of the \(S_z\) operator.

\[
S_z |\uparrow\rangle = \frac{1}{2} \hbar |\uparrow\rangle
\]

\[
S_z |\downarrow\rangle = -\frac{1}{2} \hbar |\downarrow\rangle
\]

Given the Pauli matrix \(\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\), which of the following is an eigenstate of \(S_x\) with eigenvalue \(-\frac{1}{2} \hbar\) ?

(A) \(|\downarrow\rangle\)

(B) \(\frac{1}{\sqrt{2}} (|\uparrow\rangle + |\downarrow\rangle)\)

(C) \(\frac{1}{\sqrt{2}} (|\uparrow\rangle - |\downarrow\rangle)\)

(D) \(\frac{1}{\sqrt{2}} (|\uparrow\rangle + i |\downarrow\rangle)\)

(E) \(\frac{1}{\sqrt{2}} (|\uparrow\rangle - i |\downarrow\rangle)\)