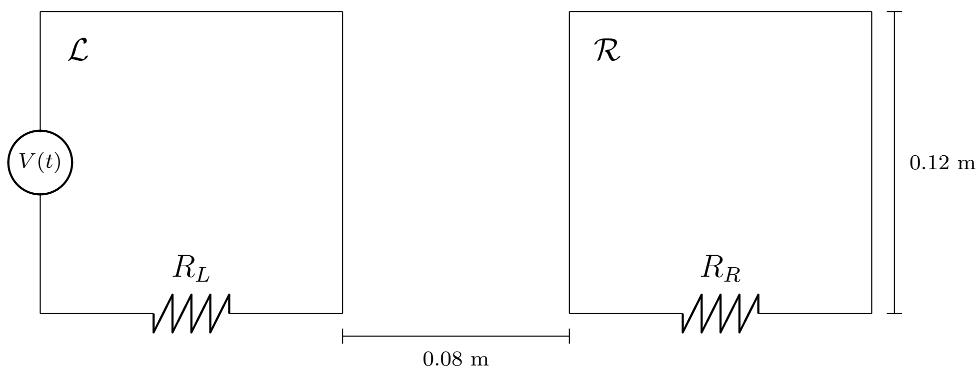
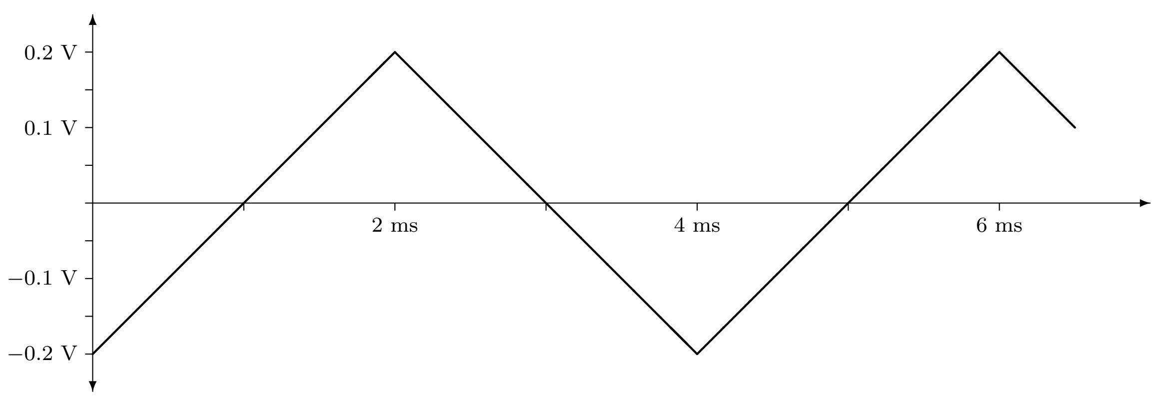
Solutions to Assignment 5; Phys 186

1. (40 points) You have two circuits next to each other:

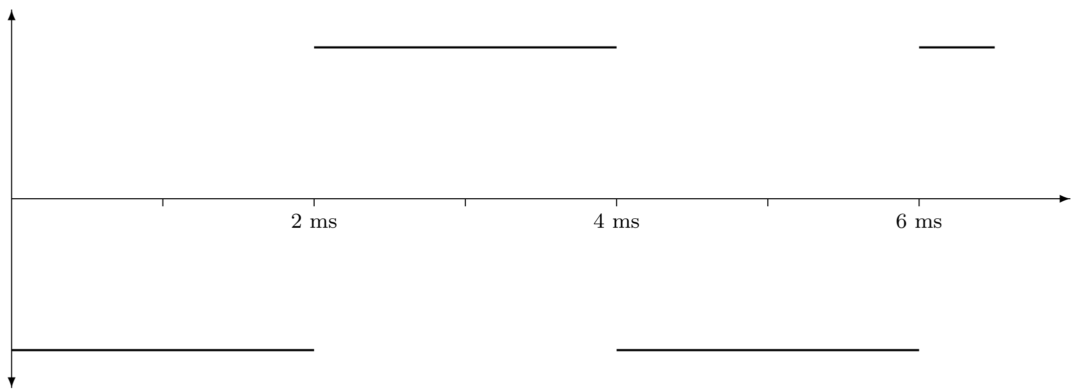


Each circuit is square with 0.12 m sides, and 0.080 m separates the right edge of the left circuit ($\mathcal{L}$) from the left edge of the right circuit ($\mathcal{R}$). Both have the same resistance values $R_L = R_R = 0.50\ \Omega$, and $\mathcal{L}$ has a voltage source which produces a sawtooth waveform $V(t)$, which looks like the following on an oscilloscope:



(a) Sketch the shape of the waveform you will see if you measure the voltage across R_R with an oscilloscope. Don't put in any voltage numbers—just sketch the waveform.

Answer: The induced voltage $V \propto -\frac{d}{dt}\Phi \propto -\frac{d}{dt}B \propto -\frac{d}{dt}I$. Therefore the shape of the waveform will go like the slope of the $V(t)$ curve:



(b) Now let's make some approximations to estimate the amplitude of the voltage waveform induced in $\mathcal{R}$. There are four wire segments in $\mathcal{L}$: the left, top, right, and bottom on the diagram. The current in each wire produces a magnetic field through $\mathcal{R}$. Only one of the following makes the largest contribution to the magnetic flux through $\mathcal{R}$—we will just take that and ignore the rest. Circle your answer:

<u>The right wire</u> The top and bottom wires The left wire

Brief explanation: The magnetic field falls off with distance; the right wire is closest to $\mathcal{R}$ and therefore will give the largest contribution.

The magnetic field produced by the wire segment you picked will not be uniform through $\mathcal{R}$. But we are looking for a rough estimate, so we will assume that is is uniform. The magnitude of B at what part of $\mathcal{R}$ will be a representative value to use in this uniform approximation?

The right edge <u>The center</u> The left edge

Brief explanation: The magnetic field falls off with distance; the center is therefore a more representative value between the right and left extremes.

Now we need an equation that will help us get the magnetic flux:

Loop: $B = \dfrac{\mu_0 I}{2r}$ <u>Long wire:</u> $B = \dfrac{\mu_0 I}{2\pi r}$ Wire: $F = ILB$

Brief explanation: The loop equation refers to the field at the *center* of a circular loop, which is not at all this situation. The force on a wire is totally irrelevant. Though the magnetic field created by a long wire is not a very accurate approximation here, since the wire length is comparable to the distance to the wire, we only need a rough estimate.

Finally, use all this and estimate the *amplitude* of the waveform sketched in part (a).

Answer: We use $r = 0.14$ m as the distance to the center of $\mathcal{R}$, and the area $A = (0.12\ \text{m})^2$. In that case

$$V = -\frac{d}{dt}\Phi = -\frac{\mu_0 A}{2\pi r}\frac{d}{dt}I$$

The current is $I = V(t)/R_L$, therefore

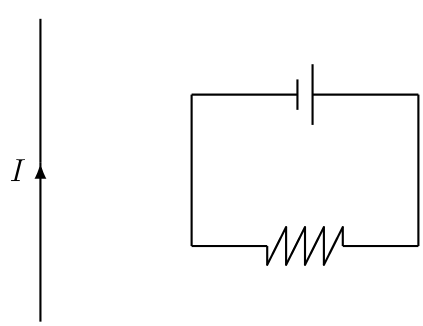
$$\frac{d}{dt}I = \frac{1}{R_L}\frac{d}{dt}V(t)$$

The rate of change of the sawtooth voltage is the slope, ± 200 V/s, depending on whether it is increasing or decreasing. The induced voltage will therefore be

$$V = \mp\frac{\mu_0 A}{2\pi r R_L}\frac{d}{dt}V = \mp 8.2 \times 10^{-6}\ \text{V}$$

So the amplitude is, very approximately, 8.2×10^{-6} V.

2. (20 points) You have a long current-carrying wire, and a circuit next to it:



(a) In the picture, draw in the magnetic field produced by the long wire with current I, and indicate the current direction in the circuit.

Answer: By the right hand rule, the magnetic field produced by the long wire will be into the page where the circuit is located. The field strength will decrease as you go father from the wire, which you can indicated by drawing $\times$ symbols less densely as you move away.

The current in the circuit will be clockwise, since the + end of the battery is on the right.

(b) Will the circuit be attracted to the long wire, repelled by it, or will it feel no force? State your reasoning.

Answer: By the right hand rule, the magnetic force on the current in the part of the circuit closest to the wire will be toward the left. The segment with the battery will feel a force upward. The far segment will feel a rightward force. And the segment with a resistor will feel a downward force.

Observing the symmetry of the situation, the upward and downward forces will be equal and opposite: they will cancel. The magnitude of the leftward force, however, will be larger than that of the rightward force. This is because while the currents are the same, the magnetic field magnitude is smaller on the side farther away from the long wire.

So when you add all the forces up, you will end up with a total force toward the left, which means that the circuit will be attracted to the wire.

(c) If you let the circuit move according to the total magnetic force that might be acting on it, will any extra voltage V_{extra} be induced in the circuit? State your reasoning.

Answer: Yes. If the circuit moves toward the left, its area and orientation won't change, but the magnetic field will become stronger. Therefore the magnetic flux Φ will change, and a non-zero rate of change will mean a non-zero V_{extra} induced in the circuit.

3. (40 points) This question is about charges in electric and magnetic fields.

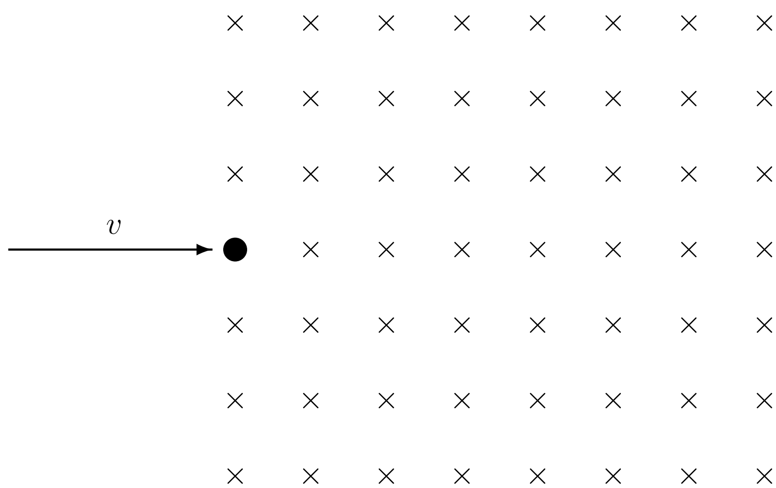
(a) Magnetic fields act on moving charges. To get electrons moving at high speed, we can accelerate them by using a high voltage. Say an electron with charge $-e$ and mass m starts at rest, and gains kinetic energy by accelerating through a voltage difference of $-V_a$. What is v, its final speed, in terms of e, m, and V_a?

Answer: The energy gained by the electron will be eV_a. This goes into kinetic energy, $\frac{1}{2}mv^2$. Solving,

$$v = \sqrt{2eV_a/m}$$

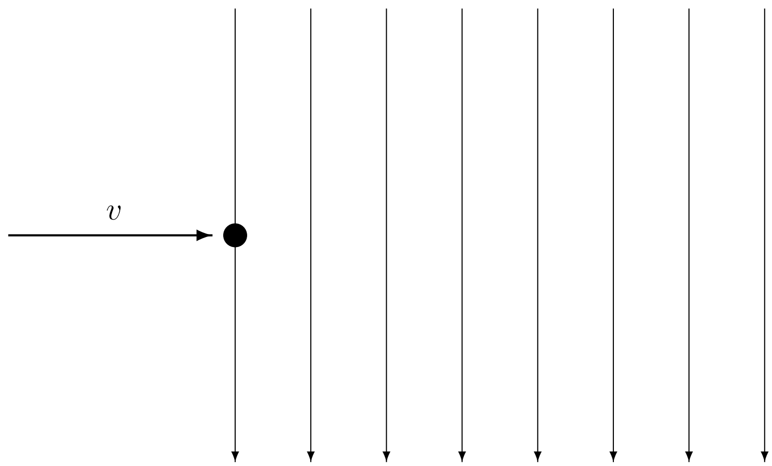
(b) An electron with speed v, moving toward the right, enters a region with a uniform magnetic field with magnitude B. The magnetic field points into the page. What is the magnitude of the magnetic force on the electron, F_B, in terms of B, e, m, and V_a? Draw its direction as the rightward-moving electron just enters the region with the uniform magnetic field into the page.

(c) Draw a diagram showing the *trajectory* of the electron in the same magnetic field. (The black dot is, once again, the electron entering the magnetic field.)



Answer: This magnetic field will produce a force $F_B = evB$, always perpendicular to the velocity. This bends the electron in a circular path. By the right-hand-rule, and the fact that the electron has a negative charge, $\vec{F}_B$ will point downward on the page. Therefore the electron bends downward in a circular arc.

(d) Now say that the electron, instead of emerging into a magnetic field, enters a region with a uniform electric field. The electron with speed v, moving toward the right, comes in between the plates of a parallel plate capacitor that produces an electric field with magnitude E. The field points downward on your page. Draw a diagram showing the trajectory of the electron in the uniform electric field.



Answer: The electric force is $F_E = eE$ in magnitude. Since the electron charge is negative, $\vec{F}_E$ points upward on the page. This *constant* force will bend the electron upwards, in a parabolic trajectory.

(e) The capacitor has a voltage difference of V_c across its plates, which are separated by a distance d. What is E, the magnitude of the electric field in this capacitor, in terms of V_c and d? **Answer:** The voltage in a capacitor increases linearly with distance, from 0 to V_c. Therefore

$$E = \frac{V_c}{d}$$

(There are multiple ways to get this.)

(f) Now say that the electron with speed v emerges into an region where *both* a uniform magnetic field and a uniform electric field exists in the ways described in (b) and (d). If the magnitude B is just right, the electric force and magnetic force will cancel each other out and the electron will not be affected. Find this value of B in terms of V_c, V_a, e, m, and d.

Answer: Setting $F_E = F_B$,

$$evB = e\frac{V_c}{d} \qquad \Rightarrow \qquad B = \frac{V_c}{vd} = \frac{V_c}{d\sqrt{2eV_a/m}}$$