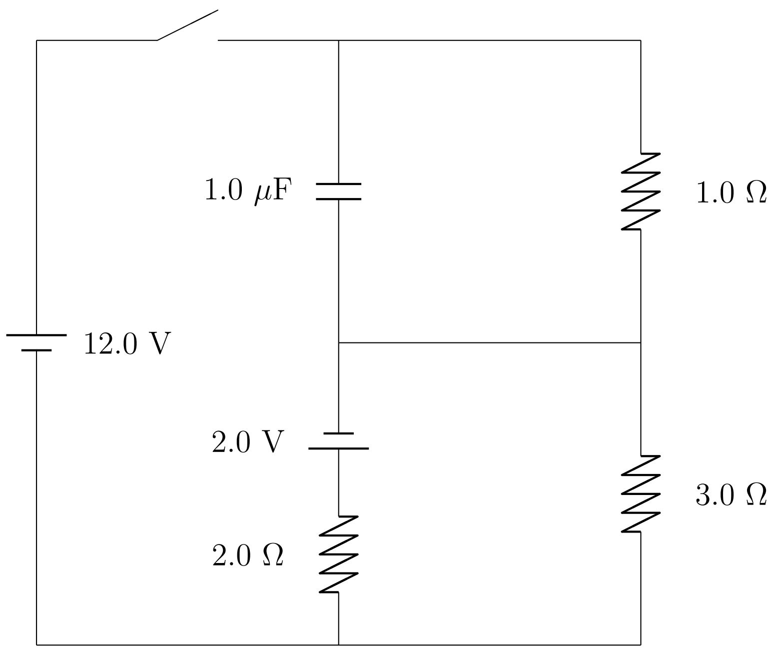
Solutions to Practice 2; Phys 186

1. (0 points) Just before the switch is closed, the capacitor in the following circuit is completely discharged. You then close the switch.



(a) Find the currents through all the resistors *immediately after* the switch is closed, before the capacitor has any time to charge up at all.

Answer: If the capacitor has had no time to build up any charge, the voltage across it must be $V = Q/C = 0$. The 1 Ω resistor is in parallel with the capacitor, hence it sees the same voltage of 0. Therefore no current flows through this resistor—all the current will go through the capacitor, bypassing this resistor. $I_1 = 0$. In other words, an uncharged capacitor behaves like a closed switch, shorting out anything in parallel to it.

This simplifies the circuit: in effect, we can replace the capacitor with a straight wire, and remove the 1 Ω resistor from the circuit. We now analyze the circuit with the two remaining resistors. The junction equation is

$$I_0 = I_2 + I_3$$

where I_0 is the current put out by the 12 V battery. The two loop equations are

$$12 \text{ V} + 2 \text{ V} = (2\ \Omega)I_2$$

$$(2\ \Omega)I_2 = 2 \text{ V} + (3\ \Omega)I_3$$

The first loop equation directly gives $I_2 = (12 + 2)/2 = 7$ A. Putting that into the second loop equation produces $I_3 = (2 \cdot 7 - 2)/3 = 4$ A.

(b) Find the currents through all the resistors *a very long time after* the switch is closed, after the capacitor has completely charged up.

Answer: After a long time, the capacitor will have fully charged, and the current through it will be zero. All the current will go through the parallel 1 Ω resistor. In other words, in this case we remove the capacitor, replacing it with an open switch. The circuit equations are very similar, with $I_0 = I_1$ now going through the 1 Ω resistor as well.

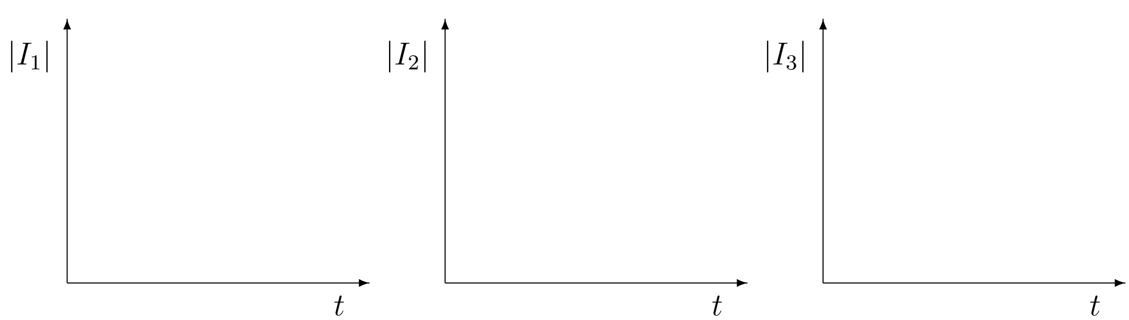
$$I_1 = I_2 + I_3$$

$$12 \text{ V} + 2 \text{ V} = (2\ \Omega)I_2 + (1\ \Omega)I_1$$

$$(2\ \Omega)I_2 = 2 \text{ V} + (3\ \Omega)I_3$$

Solving, we end up with $I_2 = 4$ A, $I_3 = 2$ A, and $I_1 = 6$ A.

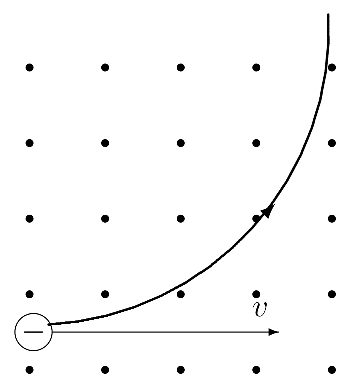
(c) Sketch qualitative I vs t graphs for each of the currents. The switch closes at $t = 0$.



Answer: I_1 will rise from 0 and level off asymptotically at 6 A. I_2 and I_3 will have exponential decay curves between their $t = 0$ and $t \to \infty$ values.

2. (0 points) Let's say you did an experiment determining the charge to mass ratio of an electron, e/m_e, using only a magnetic field. You accelerate your beam of electrons, starting from rest, through an accelerating voltage V_a, and shoot them into a region with a uniform magnetic field perpendicular to their velocity. You then measure r, the radius of the arc into which the beam is bent.

(a) In the picture below, indicate the direction of the magnetic field. To the right of the picture, briefly explain your choice. The electrons come from the left; the picture shows one electron just entering the region with the magnetic field. The arc is the trajectory the electron will follow.



Answer:

The magnetic force must be directed upwards, toward the center of the circle. Using the right hand rule, this would mean a magnetic field into the page. But an electron has a *negative* charge, so we need to reverse the direction for a magnetic field coming out of the page.

(b) Find an equation for e/m_e in terms of what you can measure or set in the lab: V_a, r (the radius of the arc), and B (the magnitude of the magnetic field). *Hint:* You may want to remind yourself about uniform circular motion from the first semester.

Answer: The only significant force on the electron is the magnetic force. (Gravity is too weak.) Therefore the magnetic force supplies the centripetal force for moving in a circle:

$$evB = \frac{mv^2}{r}$$

We need the speed v, which we can get from energy conservation:

$$\frac{1}{2}mv^2 = eV_a \qquad \Rightarrow \qquad v = \sqrt{\frac{2eV_a}{m}}$$

Putting these two together,

$$eB = \frac{m}{r}\sqrt{\frac{2eV_a}{m}} \qquad \Rightarrow \qquad \frac{e}{m} = \frac{2V_a}{B^2r^2}$$

3. (0 points) You have an appliance that draws a large current I when in operation. This current creates a magnetic field in the next room, which we measure to be roughly uniform with magnitude $B = kI$, with a constant $k = 0.027$ T/A. In the next room, you have another appliance, which we shall model as a simple circuit with resistance 14.0 Ω, which presents an area of $A_\perp = 0.13$ m^2 perpendicular to the magnetic field, and an area of $A_\parallel = 0.049$ m^2 parallel to the magnetic field.

Starting from $I = 0$, you switch your appliance on. The current rises at a constant rate for a time interval of 0.21 s, reaching a value of 18.0 A. After this interval, the current remains constant at 18.0 A.

Hint for the following: You might need to calculate the rate of change of the magnetic field. To do so, notice that k is a constant, so that $\frac{d}{dt}B = \frac{d}{dt}(kI) = k\frac{d}{dt}I$.

(a) Find the current induced in the circuit in the other room *just before the current is switched on.*

Answer: The current, and therefore the magnetic flux, is not changing before the appliance is switched on. Therefore no voltage is induced, and $I_{\text{induced}} = 0$.

(b) Find the current induced in the circuit in the other room *during the 0.21 seconds in which the current is rising.*

Answer: Since the current rises to 18 A from 0 in 0.21 s, the rate of change of current is $\frac{d}{dt}I = (18 - 0)/0.21 = 85.7$ A/s. The induced voltage is

$$V = \left|\frac{d\Phi}{dt}\right| = A_{\perp}\frac{dB}{dt} = A_{\perp}k\frac{dI}{dt} = 0.30 \text{ V}$$

This induces a current of

$$I_{\text{induced}} = \frac{V}{R} = 0.021 \text{ A}$$

(c) Find the current induced in the circuit in the other room *after the current reaches its constant* 18.0 A *value.*

Answer: Once again, since the current is not changing, Φ is not changing, so no current is induced.

(d) If your appliance worked on an AC rather than DC power supply, would you expect it to disrupt surrounding circuits differently before, during, and after you turn the power on? Explain.

Answer: While the appliance is on, an AC appliance will be creating a constantly changing Φ, since the current it draws varies sinusoidally, and quite rapidly. Therefore an AC appliance disrupts nearby circuits continually while it is in operation, not just in the process of turning on.

4. (0 points) You have a proton and an antiproton at rest on Earth. They annihilate to produce a muon-antimuon pair: $p + \bar{p} \to \mu^- + \mu^+$. The muon heads toward the Moon, 3.8×10^8 m away, and the antimuon is captured by a detector here on Earth. The typical lifetime of a muon is 2.2×10^{-6} s. Will the muon make it to the Moon to be captured by a detector there? A muon's mass is $m_\mu = 1.9 \times 10^{-28}$ kg, or 110 MeV/c^2. A proton's mass is

$m_p = 1.7 \times 10^{-27}$ kg or 940 MeV/c^2. The speed of light is 3.0×10^8 m/s. Note:

- Relativistic energy (γmc^2) and momentum ($\gamma m\vec{v}$) are both conserved in this reaction. Show how you use both.
- You'll get a **bonus +5 points** if you solve this using the masses given in MeV/c^2.

Answer: The masses of particles and their antiparticles are identical. Therefore, with the proton and antiproton starting from rest, momentum conservation looks like

$$0 = \gamma_{\mu^-} m_\mu \vec{v}_{\mu^-} + \gamma_{\mu^+} m_\mu \vec{v}_{\mu^+}$$

This implies the velocities of the muon and antimuon are equal and opposite, and that therefore $\gamma_{\mu^-} = \gamma_{\mu^+}$.

Energy conservation then gives, with $\gamma_p = \gamma_{\bar{p}} = 1$ because they are at rest,

$$2m_pc^2 = 2\gamma_\mu m_\mu c^2 \qquad \Rightarrow \qquad \gamma_\mu = \frac{m_p}{m_\mu} = 8.6$$

This is a large γ, which implies the speed of the muon $v \approx c$. In the frame of reference of an observer on Earth, the muon's lifetime will increase due to time dilation; it will live for $\Delta t = \gamma_\mu (2.2 \times 10^{-6}\text{ s})$. During this time it will travel close to the speed of light, covering the proper distance

$$\Delta x_0 = c\gamma_\mu (2.2 \times 10^{-6}\text{ s}) = 5.6 \times 10^3\text{ m} < 3.8 \times 10^8\text{ m}$$

The muon will fall far short of the Moon.