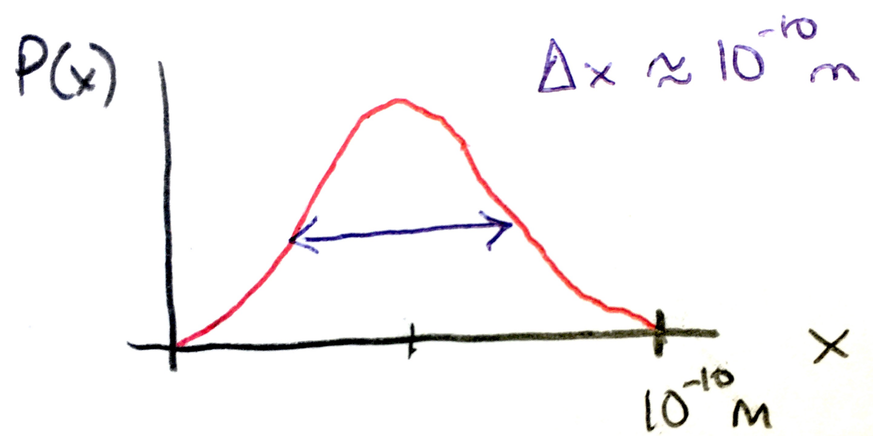
Solutions to Exam 2; Phys 100

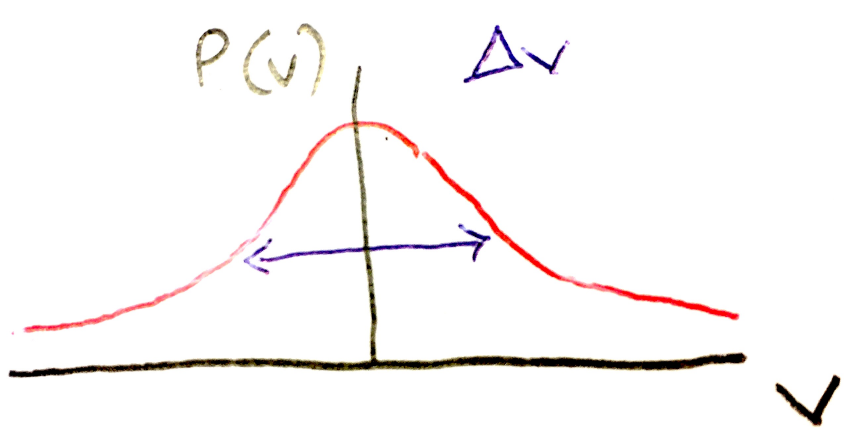
1. (50 points) Say you have an electron that is free to move in one dimension (call it the x-axis), but its location is confined to be between $x = 0$ and $x = 10^{-10}$ m. In other words, if you measure the location x of the particle, it is *impossible* to find an x that is negative or is greater than 10^{-10} m.

(a) Draw a simple, single-peak probability distribution $P(x)$ for the location of the particle, such that the average value for measured locations will be $x = 0.5 \times 10^{-10}$ m. Also indicate the width of the distribution, and write down an estimate of this width.

Answer:



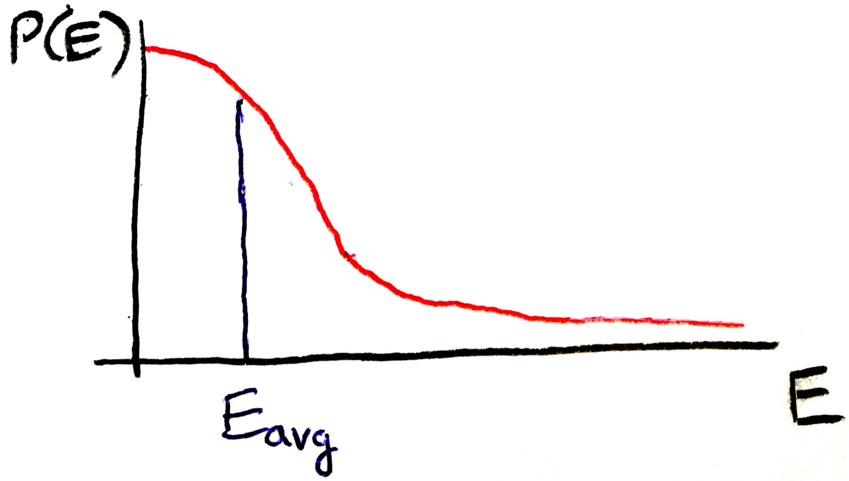
(b) Draw a simple, single-peak probability distribution $P(v)$ for the velocity of the particle, such that the average value for measured velocities will be $v = 0$. (Remember that velocity is speed plus direction, and that the direction is indicated by positive or negative signs!) Assuming that this distribution has the *minimum possible* width given your answer to (a), calculate this width and indicate this on your drawing.



Answer:

The width Δv can be obtained from the uncertainty principle. Since it's the minimum possible uncertainty, $\Delta v = h/(4\pi m_e \Delta x) = 5.79 \times 10^5$ m/s. This is not a small uncertainty.

(c) The energy of the electron is purely kinetic energy, which is proportional to the *square* of the velocity. Draw the probability distribution $P(E)$ for the energy E of the particle. Will the average energy be negative, zero, or positive? *Hint:* Will it be possible to have negative kinetic energies for the particle?



Answer:

Since the square of the velocity can't be negative, the energy can't be negative. And unless the energy is certain to be zero, the average energy has to be positive.

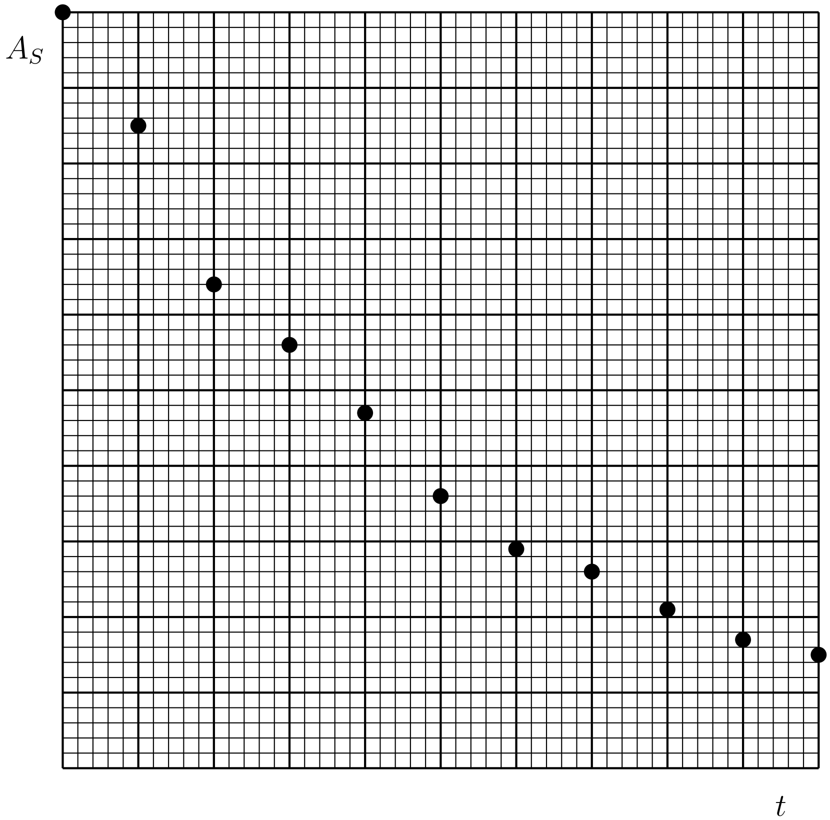
(d) If you confined a marble to bounce between two walls it would eventually settle down and have zero velocity and zero kinetic energy. Given your answers, is this what you would expect for a confined electron, which has to be described by quantum mechanics?

Answer: Zero velocity would contradict the uncertainty principle, as the previous answers show. A confined electron cannot have zero energy.

2. (30 points) You want to measure the half-life of a radioactive sample. You first measure the background radiation in your lab: 23 counts per minute. Then, every day at 12:00 noon, you take a detector and count the total number of events with your sample in place for exactly one minute. Call this activity A_T. Here is a table of your data:

t (days)	0	1	2	3	4	5	6	7	8	9	10
A_T (counts/minute)	123	108	87	79	70	59	52	49	44	40	38
A_S (counts/minute)	100	85	64	56	47	36	29	26	21	17	15

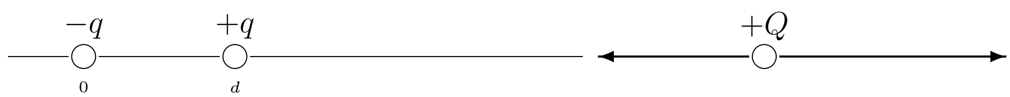
(a) Make a graph of A_S, *the activity due to your sample alone*, and time. Clearly label the axes of your graph.



(b) Using your graph, estimate the half-life of your radioactive sample. Explain your reasoning.

Answer: Looking at your graph, it looks like going down to 50 counts/minute, which is half the initial activity of 100 counts/minute, happens around 3.6 days. If that's the half-life, if we wait another 3.6 days, at 7.2 days, we should get around 25 counts/minute. That seems about right from the graph. So, 3.6 days is the half-life.

3. (40 points) Atoms and molecules are combinations of electrically charged subatomic particles that are electrically neutral overall. Consider a simple model: a combination of a positive $+q$ charge and a negative $-q$ that are fixed such that the distance between them is d. There is also another charge $+Q$ that is further away.



(a) Draw an arrow, with its base on the charge $+Q$, indicating the electrical force the $+Q$ feels due to the $-q$ charge. Then draw another arrow indicating the electrical force the $+Q$ feels due to the $+q$ charge. If one of these forces is stronger than the other, make sure that its arrow is longer; if they are equal, make sure that they are visibly equal in length.

(b) Consider the total force on $+Q$ due to the combination of $+q$ and $-q$ on the left. Is this total force zero, toward the right, or toward the left? If your answer is zero, is this because the total charge of the combination of $+q$ and $-q$ charges is zero? If your answer is non-zero, explain how this electrically neutral combination of charges can exert an electric force.

Answer: The $+q$ charge is closer to the $+Q$ charge, and therefore the repulsive force on $+Q$ is stronger. The total force on it will be repulsive, toward the right.

The combination of $+q$ and $-q$ is neutral, but it still applies a residual electrical force because the charges are not exactly on top of each other. And external charge such as $+Q$ ends up feeling a force, depending on which end it happens to be closer to.

(c) Water molecules, which are electrically neutral, behave like fixed combinations of $+q$ and $-q$ charges. At low temperatures, water is liquid, even solid. Given this, and your answers to (a) and (b), can you conclude that fixed combinations of $+q$ and $-q$ charges attract one another, repel one another, or don't interact at all?

Answer: To make a liquid or solid, the water molecules must adhere together. In other words, they must apply attractive forces to one another.

(d) A proton is a combination of three quarks, uud, that is neutral with respect to the color charge, such as RGB. The strong force is mediated by an exchange of gluons. Gluons interact with particles that have a non-neutral color charge. But if a proton is color-neutral, how can it possibly interact with another proton through the strong force, since both are color-neutral combinations of three quarks?

Answer: Just like neutral combinations of electric charges still exert residual electric forces, neutral combinations of color charge can exert a residual strong force, because the quarks that make up a proton are not exactly on top of each other. A quark in a neighboring proton will feel a force, depending on what it happens to be closest to.

The strong force that keeps protons and neutrons bound in the nucleus turns out to be a mere residue of the even stronger force between quarks.

4. (30 points) In pop culture and on web sites that try to sell you "energy healing" and the likes, you will often run into proclamations like "If you go to quantum physics, we realize everything is energy," or "Quantum mechanics reveals that your perception determines the shape of your reality." In class, I warned you that many such quantum references were nonsense. But I also told you that physicists don't know everything: for example, we do not know what dark matter and dark energy consist of, and they make up 95% of our universe. What if someone who claims to perform magical feats of healing attributes their magic to quantum physics, and when physics professors object, just add that their healing powers are realities that mainstream physics has not caught up with yet? How would you respond? How does what you have learned about quantum physics in this course help you evaluate such magical claims?

Answer: Answers will vary. But I hope you will notice that:

- Just because we know little about certain things in physics, such as dark matter and dark energy, does not mean we're equally in the dark (pun intended) about other things. We've got basic quantum mechanics nailed down pretty well.
- It's certainly *possible* that physicists have it wrong when we dismiss quantum healing claims. Maybe there is something we don't know yet. But how *plausible* is such a claim? How likely is it that physicists have made such a big error?