College Physics I
Lab 3: Acceleration Due to Gravity

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This is the first lab in which we will actually measure something—the acceleration due to gravity near the surface of the earth: $g$. We will first calculate what, theoretically, the acceleration should be for a cart being pulled by a known force. Then we will determine that same acceleration experimentally. By comparing our theoretical expression with our experimental result, we will be able to obtain a value for $g$.

Setup

Connect two light gates in “daisy chain” (back to back) mode, and hook them up to the Green Vernier device. (Consult the example setup I’ll have in the lab.) Then open Logger Pro. You will see four windows: a data table, and graphs for position, velocity, and acceleration. We will use only the timing column on the table. Disregard all the rest.

Play with the gates. Blocking the path of light between its ends will switch the state of the gate (0 or 1). By looking at the timing column, you can figure out the time elapsed, $\Delta t$, between when a cart passes through one gate and then the other. You will need to mount something like a card on top of your carts, using tape, to make it block the light gates as it goes by.

Make sure you understand what your equipment is doing, and decide which two of the four timing numbers this setup will produce you will use to get $\Delta t$. 
Acceleration of a cart pulled by a hanging mass

If a cart is attached to a hanging mass as shown, then the force of gravity on the hanging mass should cause both the hanging mass and the cart to accelerate. The magnitude of the total force exerted by the hanging mass is $m_{\text{hanging}}g$, and this must equal the acceleration of the total mass of $m_{\text{total}} = m_{\text{hanging}} + m_{\text{cart}}$. Using this information, we can derive an expression for the acceleration of the cart, which is the same as the acceleration of the total mass since they are all connected.

$$F = m_{\text{total}}a_{\text{theoretical}},$$
$$F = m_{\text{hanging}}g,$$
$$m_{\text{total}} = m_{\text{hanging}} + m_{\text{cart}},$$
$$\Rightarrow a_{\text{theoretical}} = \frac{m_{\text{hanging}}g}{m_{\text{hanging}} + m_{\text{cart}}}. \quad (1)$$

Applying \textit{force} = \textit{mass} \times \textit{acceleration} to this situation, this is the predicted acceleration of the cart. We have not yet done the experiment. Therefore, we call this expression for the expected acceleration $a_{\text{theoretical}}$. Note that everything on the right hand side of Equation (1), except for $g$, is either directly measurable or given. Furthermore, note that nothing on the right side of Equation (1) depends on actually doing the experiment (that is, actually letting the cart accelerate).
G AS A FUNCTION OF THINGS WE CAN MEASURE

Acceleration of the cart

As is often the case, we are not able to directly measure what we want, the acceleration of the cart. Instead, we make other measurements and obtain the acceleration from those measurements. Using the markings on the track, we can find \( x_f \) and \( x_i \), and using the light gate we can find the time \( \Delta t \) it took to travel that distance. The total distance covered is \( \Delta x = x_f - x_i \). Thus we can find the average velocity, \( \bar{v} = \Delta x / \Delta t \).

Assume that the cart starts at rest (\( v_i = 0 \)), and assume that the acceleration (whatever it turns out to be) is constant. Then in time \( \Delta t \) the cart will reach a maximum speed (\( v_f \)) equal to twice \( \bar{v} \). (Ask yourself: why twice?) But we know that \( v_f \) must equal \( a_{\text{experimental}} \Delta t \), so we can derive an expression for \( a_{\text{experimental}} \):

\[
a_{\text{experimental}} = \frac{v_f - v_i}{\Delta t},
\]
\[
v_f = 2\bar{v},
\]
\[
\bar{v} = \frac{\Delta x}{\Delta t},
\]
\[
\Rightarrow a_{\text{experimental}} = \frac{2\Delta x}{(\Delta t)^2}. \tag{2}
\]

Note that everything on the far right side of Equation (2) is directly measurable. Note, also, that in this derivation we have assumed that the starting velocity \( v_i = 0 \). If that weren’t the case, then \( v_f \) would not be equal to twice \( \bar{v} \). Thus, in the experiment coming up, we should be sure that the speed of the cart really is zero when we start the timer. Otherwise, Equation (2) does not apply!

**g as a function of things we can measure**

We expect that the theoretical value should equal the experimental value. Therefore we can combine Equations (1) and (2) and solve for \( g \):

\[
\frac{a_{\text{theoretical}}}{m_{\text{hanging}} g} = \frac{a_{\text{experimental}}}{(m_{\text{hanging}} + m_{\text{cart}}) \Delta t^2},
\]
\[
\Rightarrow g = \frac{2(m_{\text{hanging}} + m_{\text{cart}}) \Delta x}{m_{\text{hanging}} (\Delta t)^2}. \tag{3}
\]
PART 1: AN EXPERIMENT TO FIND $g$

Thus, by letting a cart accelerate and carefully measuring the various masses, positions and times, we can determine $g$.

This is a good place to stop and think for a minute. Look at Equation (3), and notice that the precision of our result depends on the precision of our measurements. Furthermore, a sloppy job in taking just one of the measurements will ruin the whole result. Thus, when doing an experiment, you must do every step as carefully as you possibly can.

Part 1: An experiment to find $g$

Using the a cart and hanging mass, do an experiment, using light gates and the set-up shown in Figure 1, in which you find $g$. Make your hanging mass at least 20 g. More would be fine. If the hanging mass is too small, the force pulling the cart becomes too small compared to the force of friction in the cart’s wheel bearings. Also pay attention to the length of the string—too long and the hanging mass will hit the ground too soon, too short and the distance traveled will be too small.

Your raw data will consist of $m_{\text{cart}}$, $m_{\text{hanging}}$, $\Delta x$, and $\Delta t$. From those data, using Equation (3), you can calculate $g$. Do at least 2 different experiments (the more the better). In other words, change at least one of $\Delta x$, $m_{\text{cart}}$ and/or $m_{\text{hanging}}$, and see if you get the same (or close to the same) value of $g$ for each one.

Be careful: You need to make sure $v_i = 0$ as the cart goes through the first photogate!

How many significant figures should you give for your final result? Your calculator can probably give you something to 8 or more figures, but are all of those figures meaningful? To answer that question, you must look at your measurements. To how many figures can you meaningfully report $m_{\text{cart}} + m_{\text{hanging}}$? How about $\Delta x$? How about $m_{\text{hanging}}$ and $\Delta t$? All of those must be considered, and the one which is the “weakest link” (that is, the one with the fewest significant figures) determines the number of significant figures in your final result.

Being aware of this issue of precision may actually affect how you choose to do the experiment. As an example, if the distance between the starting and stopping points is very small, then $\Delta x$ will be very small, and you will not be able to determine it very precisely. But if you make that distance very big, then even a larger imprecision in your position measurement will
PART 2: COMPARING YOUR RESULT TO THE ACCEPTED VALUE

not affect $\Delta x$ very much.

(Note: There are more accurate ways to deal with the fact that the $\Delta t$ is squared in determining the number of significant figures for our final result. For our purposes, it is sufficient to just concern yourself with the number of significant figures in $\Delta t$, not $(\Delta t)^2$. In general, in this course I won’t require you to pay detailed attention to significant figures. Just don’t get carried away and write down everything your calculator spits out at you.)

To hand in for part 1

- Your explanation of your choice of two of the four timing numbers.
- All raw measurements,
- Equation used to find $g$,
- Final results for $g$.

Part 2: Comparing your result to the accepted value

The accepted value of $g$ is, to 3 significant figures, 9.80 m/s$^2$. If your result is good to 3 significant figures, than that is what you should compare it to; if your result is only good to two significant figures, then you should just compare it to 9.8 m/s$^2$, and so on. A common way to compare a value to an accepted value is the percent error:

$$\text{percent error} = \left| \frac{\text{accepted value} - \text{measured value}}{\text{accepted value}} \right| \times 100\%.$$  

It is also useful to compare your results with those of at least one other lab group. You are doing the same experiment, so you should be getting similar results. Your experiments will not and should not all turn out perfectly, because of the limitations of your tools and because of the assumptions that go into our derivations. However, I will expect you to at least notice that there is a mistake. Try to find and correct all mistakes, and ask me for help if it is necessary.
PART 2: COMPARING YOUR RESULT TO THE ACCEPTED VALUE

Something to keep in mind when comparing results of experiments to accepted values is: What are the assumptions that went into this result, and how good are they? For example, in this experiment, we assumed that the cart rolled frictionlessly on the track, that the track was perfectly level, and that the acceleration of the cart was constant. To the degree that any of those assumptions are not really correct, your result should not agree with the accepted value! This is another factor that influences how you do the experiment—the more carefully you level the track, for example, the more correct your assumption of a level track is.

To hand in for part 2

- Quantitative comparison of your result for $g$ with the accepted value,
- Qualitative comparison of your result for $g$ with that of at least one other lab group.