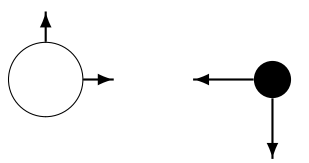
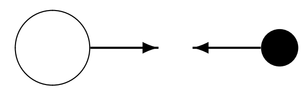
# Solutions to Assignment 3; PHYS 185

**1. (10 points)** The diagrams below show a binary star system. The white star is is more massive than the one shown as a black circle, but not hugely more massive. The stars revolve around their common center of mass. On diagram (a), draw and label the velocity and acceleration vectors for each star. On (b), show the forces on each star. Assume that the stars are in deep space and we can ignore the effects of the rest of the universe on either star. Draw the sizes of your arrows such that I can tell whether $v$, $a$, and $F$ for each star is larger, smaller, or equal to the other.

(a)



(b)



Explain your reasoning:

**Answer:** In a circular orbit, the acceleration and velocity will be perpendicular. Both stars rotate about their common center of mass.

The forces are just gravitational attraction, directed toward each other. Since the forces on the stars are action-reaction pairs, they are equal in magnitude; their arrows should be the same length.

The acceleration arrow on the less massive star should be longer. Since $\vec{a} = \sum \vec{F}/m$, and since the forces have the same magnitude, the smaller mass will have a larger acceleration magnitude.

The velocity arrow on the less massive star should be longer. The acceleration of each star is the centripetal acceleration for their circular orbits; $a_c = mv^2/r$. This means that $v^2 = ra_c$. We've already established that the magnitude $a_c$ is larger for the smaller star. The smaller star is also farther away from the center of mass, so its $r$ is larger. Therefore the magnitude $v$ is larger for the less massive star.

**2. (40 points)** You have a book sliding on a table, and the coefficient of kinetic friction between the book and the table is $\mu_k$. The table makes an angle of $\phi$ with the *vertical* (not the horizontal!). $\phi$ is small enough that if you placed the book at rest on the table, it would slide down. You now launch the book up the slope of the table with an initial speed $v_0$.

(a) What distance along the table will the book travel before it starts sliding back down?

**Answer:** Using the usual tilted coordinate axes,

$$\sum F_y = n - mg\sin\phi = 0 \qquad \Rightarrow \qquad n = mg\sin\phi$$

$$\sum F_x = -mg\cos\phi - \mu_k n = ma_x \qquad \Rightarrow \qquad a_x = -g(\cos\phi + \mu_k \sin\phi)$$

Now we do motion with constant acceleration. To come to a rest,

$$v_x = v_{0x} + a_x t = 0 \qquad \Rightarrow \qquad t = \frac{v_i}{g(\cos\phi + \mu_k \sin\phi)}$$

Choosing to call the point where the book starts $x_0 = 0$,

$$x = v_{0x}t + \frac{1}{2}a_x t^2 = \frac{v_i^2}{2g(\cos\phi + \mu_k \sin\phi)}$$

(b) The book now slides down. Find its velocity as it passes the point it was launched from. (In other words, the book slides down the same distance it traveled up in part a).

**Answer:** On the way down, the friction force reverses direction, but otherwise everything remains the same.

$$\sum F_y = n - mg\sin\phi = 0 \qquad \Rightarrow \qquad n = mg\sin\phi$$

$$\sum F_x = -mg\cos\phi + \mu_k n = ma_x \qquad \Rightarrow \qquad a_x = -g(\cos\phi - \mu_k \sin\phi)$$

Using $x$ from part (a) as the new $x_0$, and looking to see what happens when the book goes through $x = 0$ again, we get

$$0 = \frac{v_i^2}{2g(\cos\phi + \mu_k \sin\phi)} + \frac{1}{2}a_x t^2 \qquad \Rightarrow \qquad t = \frac{v_i}{g\sqrt{(\cos\phi + \mu_k \sin\phi)(\cos\phi - \mu_k \sin\phi)}}$$

$$v_x = a_x t = -v_{0x}\sqrt{\frac{\cos\phi - \mu_k \sin\phi}{\cos\phi + \mu_k \sin\phi}}$$

**3. (30 points)** You drop a spherical 4.1 kg object from a bridge with height 84 m over the water. Take $C_D = 0.5$, the density of air $\rho = 1.22$ kg/m$^3$, and the radius of the object to be 0.25 m.

(a) Assuming you can ignore air resistance, calculate the speed at which the sphere will hit the water, and the time it will take to reach the water.

**Answer:** Free fall starting from rest: $0 = y_0 - \frac{1}{2}gt^2$, therefore the time it takes to reach the water is $t = \sqrt{2y_0/g} = 4.1\,\text{s}$. The velocity will be $v_y = -gt = -41\,\text{m/s}$, for a speed of 41 m/s.

(b) But we really can't ignore the air resistance. Calculating the motion by accounting for drag force requires calculus, so we won't do that. Instead, circle the most accurate *approximate model* from amongst the following options:

**i.** Assume constant acceleration of $g$ until the sphere reaches terminal velocity, and afterwards have it continue falling with this constant terminal velocity.

**ii.** Assume a constant velocity equal to the terminal velocity downward throughout the fall.

**iii.** Assume constant acceleration throughout, but reduce the downward acceleration to $g - \frac{1}{2}C_D m$, where $m$ is the mass, to account for the drag force.

**iv.** Assume constant acceleration throughout, but reduce the acceleration to $g\frac{1}{2}C_D$, to account for the effect of the drag force.

Explain why you think your choice is the best approximation among these options.

**Answer:** Option **(i)** is best. First, eliminate **(iii)** and **(iv)**. These are meaningless; **(iii)** attempts to subtract mass from acceleration, and **(iv)** is simply strange. Among the remaining two, the first does a better job accounting for the fact that the object has to speed up to achieve terminal velocity.

(c) Use your choice of approximate model to calculate the speed at which the sphere will hit the water, and the time it will take to reach the water. Compare these to your answers from (a)—do they make sense?

**Answer:** The terminal velocity is achieved when $\frac{1}{2}C_D\rho A v^2 = mg$, therefore $v_T = \sqrt{2mg/C_D\rho A} = 26$ m/s. ($A$, here, is the cross-sectional area of the sphere, $\pi r^2$.) If we accelerate at $g$ to reach this speed, we need, from $v_T = gt$, $t = v_T/g = 2.6$ s. During this time, the object will have fallen $\frac{1}{2}gt^2 = 34$ m.

The final $84 - 34 = 50$ m will be at constant velocity $v_T$. This takes $50/26 = 1.9\,\text{s}$.

So in this model, the whole fall takes $2.6 + 1.9 = 4.5\,\text{s}$, and the sphere hits the water at a speed of $v_T = 26\,\text{m/s}$. This is $|4.5 - 4.1|/4.1 = 10\%$ longer, and $|26 - 41|/41 = 37\%$ slower than (a).

**4. (20 points)** You set up a device where you have a bucket tethered to a rope, and the bucket rotates at a constant speed $v$ in a vertical circle with radius $r$. You then place a small ball with mass $m$ in the bucket. What is the minimum $v$ you must have in order for the ball not to fall out of the bucket during rotation?

**Answer:** At the top of the circle, if the speed is just barely enough to keep the ball from falling, the normal force on the ball will be zero. The acceleration will need to be directed downwards (toward the center of the circle), and have a magnitude of $v^2/r$. Therefore

$$\sum F_y = ma_y \qquad \Rightarrow \qquad w + n = \frac{mv^2}{r}$$

With $n = 0$ for $v = v_{\mathsf{min}}$,

$$mg = \frac{mv_{\mathsf{min}}^2}{r} \qquad \Rightarrow \qquad v_{\mathsf{min}} = \sqrt{gr}$$