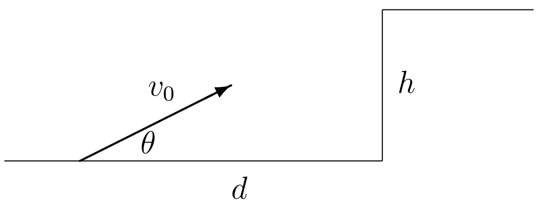
Solutions to Exam 1; Phys 185

1. (20 points) You're on a moon with no atmosphere, and acceleration due to gravity q. You have a cannon that can shoot a ball out beyond the crater wall with height h a distance d away. The angle of the cannon with the horizontal is θ, and the initial speed of the ball is v_0.



(a) Find an inequality describing the conditions under which the ball will make it out of the crater.

Answer: No atmosphere: no drag. The equations for free fall should work very well. First start by figuring out when the ball just makes it to the edge of the crater, $x = d$

$$d = 0 + v_0 \cos\theta\, t \qquad \Rightarrow \qquad t = \frac{d}{v_0 \cos\theta}$$

At that time,

$$y = 0 + v_0 \sin\theta\, t - \frac{1}{2} q\, t^2 \qquad \Rightarrow \qquad y = d \tan\theta - \frac{q\, d^2}{2 v_0^2 \cos^2\theta}$$

The condition to make it out is $y \geq h$. Therefore, the inequality we need is

$$d \tan\theta - \frac{q\, d^2}{2 v_0^2 \cos^2\theta} \geq h$$

(b) Using your inequality, you should find that there is an angle below which the ball will never make it out, no matter how large v_0 is. Find this angle. Then explain what your result means—physically, what is going on?

Answer: When v_0 becomes very large, the second term on the left side of the inequality becomes very small. (For those of you who remember limits: it goes to zero.) Therefore, the inequality becomes

$$d \tan\theta \geq h$$

If, therefore, the angle is such that

$$\tan\theta < \frac{h}{d} \qquad \text{or} \qquad \theta < \tan^{-1}\frac{h}{d}$$

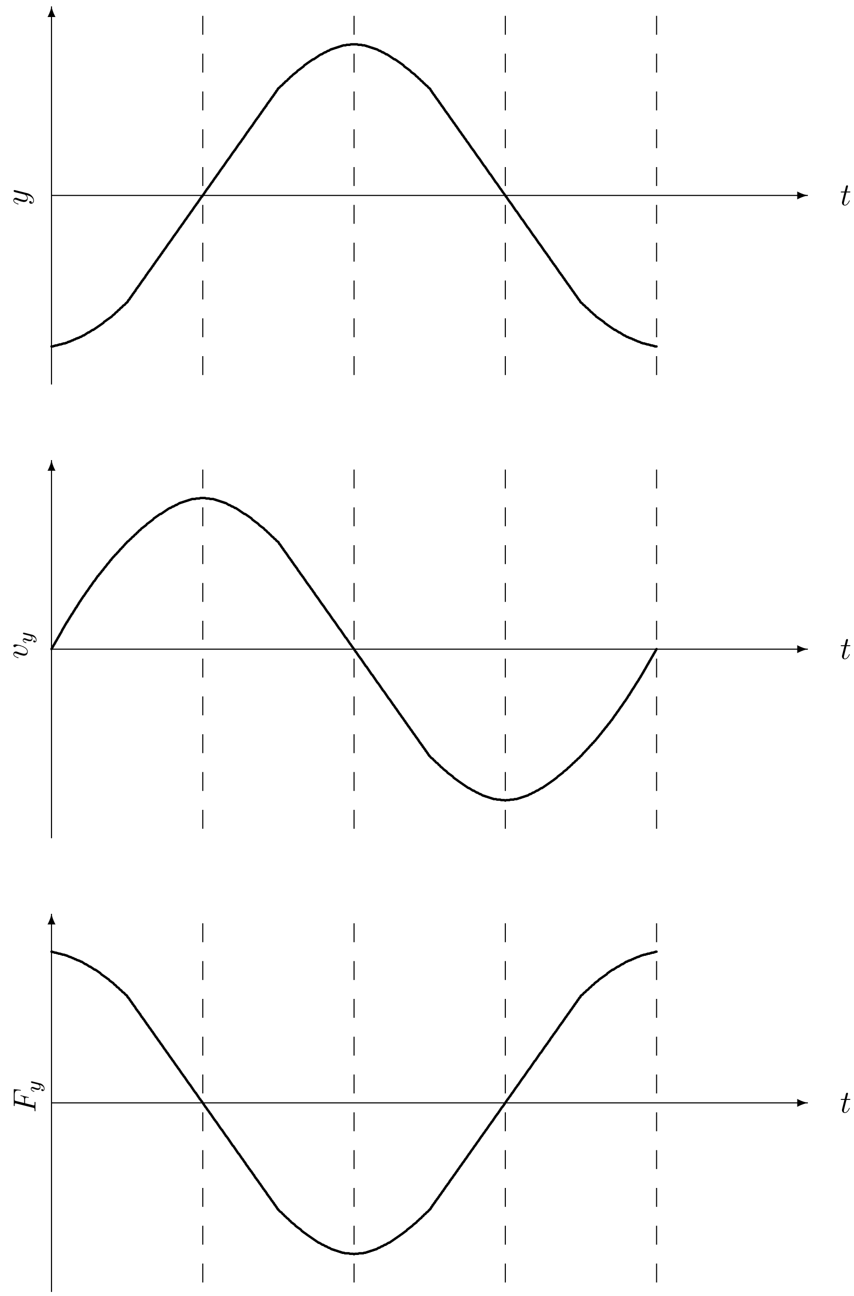
the ball can never make it out. Notice that this angle describes a triangle that connects the launch point to the lip of the crater by a straight line. At that angle, only a ball launched at infinite speed, which then will have no time to dip down under the influence of gravity, can barely make it out of the crater. Even infinite speed won't do it for smaller angles.

2. (20 points) The last graph shows the force on an oscillating mass attached to a spring (as in Lab 5) over a full period. At $t = 0$, the spring is stretched to its maximum extent downward; you let go and the mass moves upward. $y = 0$ is the position for an unextended spring. Sketch the position and velocity graphs as accurately as you can.

Answer: Force is proportional to acceleration, and therefore the acceleration graph will have the exact same shape as the given force graph. Then you just work up to the velocity graph by seeing how the area under the curve changes. You do the same to go from velocity to position.

Some points may be confusing. For example, after a quarter of the period, F_y becomes negative, and the area that you add starts getting negative. You may not know how to deal with this. But you can figure it out. After all, you've played with the spring in the lab; you know how the velocity and position of the mass behaves.

There's an extra clue for the position. Since $F_y = -ky$, the position graph should look like an inverted force graph.



3. (20 points) Say you drop a metal sphere with mass m under water, starting from rest. The drag force in water is $\vec{D} = -c\,\vec{v}$, with c a constant. Aside from drag and weight, it also has an upward buoyant force $\vec{F}_B$ acting on it, which has magnitude $F_B = m_w g$, where m_w is the mass of water with a volume equal to that of the metal sphere, and $m_w < m$. Would the sphere attain a terminal velocity, just as in air? If you think it will, derive an equation for v_T in terms of c, m, m_w, and g. Otherwise, if you think it won't, draw a rough graph of v vs t for the sphere falling in water.

Answer: Yes, the same reasoning for terminal speed in air applies. When the sphere is released, it's at rest, therefore the drag on it is zero. It accelerates downward. The weight and buoyant forces, however, are constant, and as the sphere speeds up, the drag on it increases, therefore decreasing the total downward force and making the acceleration approach zero. Terminal speed happens when all the forces cancel each other out:

$$\sum F_y = D - w + F_B = cv_T - mg + m_w g = 0 \qquad \Rightarrow \qquad v_T = \frac{(m - m_w)g}{c}$$

4. (20 points) As in your Assignment 3, you slide a book up a table. The coefficient of kinetic friction between the book and the table is μ_k, and the coefficient of static friction is μ_s. The table makes an angle of ϕ with the *vertical* (not the horizontal!). You launch the book up the slope of the table with an initial speed v_0. Is it possible that the book will get stuck at its topmost point, and not slide down again? If it is impossible, demonstrate why. If it is possible, find the conditions under which the book will get stuck.

Answer: If the angle ϕ is large enough, static friction at the point where the book would turn around (and where $v_x = 0$) can be strong enough to prevent the book from sliding down.

So, the force components will be just as in Assignment 3, but with kinetic friction replaced by static friction. For the minimum angle ϕ, set f_s to its maximum value, $f_s = \mu_s n$. We then have, using the usual tilted coordinate axes,

$$\sum F_y = n - mg\sin\phi = 0 \qquad \Rightarrow \qquad n = mg\sin\phi$$

$$\sum F_x = -mg\cos\phi + \mu_s n = ma_x \qquad \Rightarrow \qquad a_x = -g(\cos\phi - \mu_s \sin\phi)$$

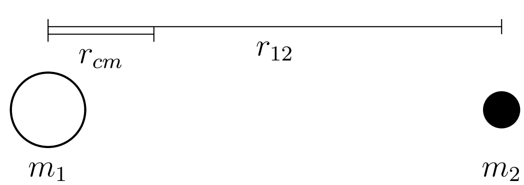
But since the book remains at rest, $a_x = 0$. Therefore, canceling out g,

$$\mu_s = \frac{\cos\phi}{\sin\phi}$$

That was the lower limit. The condition we're looking for is therefore

$$\phi \geq \cot^{-1}\mu_s$$

5. (20 points) The diagram below shows a binary star system. The white star has mass m_1, the black star mass m_2. The distance between the centers of stars is r_{12}. The only force on each star is the gravity from the other star. You look up the center of mass for two bodies and find that the center of mass is located at a distance $r_{cm} = \frac{m_2}{m_1+m_2} r_{12}$ from the center of the white star.



(a) Find equations for a_1, a_2, v_1, v_2—the magnitudes of the accelerations of each star and the speeds of each star, in terms of m_1, m_2, r_{12}, and appropriate physical constants.

Answer: The force of attraction between the stars is $F_G = G\, m_1 m_2 / r_{12}^2$. The accelerations are just that force divided by each mass:

$$a_1 = G\,\frac{m_2}{r_{12}^2} \qquad a_2 = G\,\frac{m_1}{r_{12}^2}$$

For circular motion, $a = v^2/r$, and so $v = \sqrt{ar}$. Therefore

$$v_1 = \sqrt{G\,\frac{m_2}{r_{12}^2}\, r_{cm}} = \sqrt{G\,\frac{m_2}{r_{12}^2}\,\frac{m_2}{m_1+m_2}\, r_{12}} = \sqrt{\frac{G\, m_2^2}{r_{12}(m_1+m_2)}}$$

$$v_2 = \sqrt{G\,\frac{m_1}{r_{12}^2}\,(r_{12} - r_{cm})} = \sqrt{G\,\frac{m_1}{r_{12}^2}\,\frac{m_1}{m_1+m_2}\, r_{12}} = \sqrt{\frac{G\, m_1^2}{r_{12}(m_1+m_2)}}$$

(b) To confirm your results, show that the period of the stars—the time it takes for each to make one full circle—is the same.

Answer: The periods being the same means that $2\pi r/v$ for each orbit is the same. Therefore,

$$\frac{2\pi r_{cm}}{v_1} = \frac{2\pi(r_{12} - r_{cm})}{v_2} \qquad \Rightarrow \qquad \frac{m_2}{m_1+m_2}\frac{1}{v_1} = \frac{m_1}{m_1+m_2}\frac{1}{v_2} \qquad \Rightarrow \qquad \frac{v_1}{v_2} = \frac{m_2}{m_1}$$

If you take the ratio of the answers from (a), we get

$$\frac{v_1}{v_2} = \sqrt{\frac{m_2^2}{m_1^2}} = \frac{m_2}{m_1}$$

So it checks out.