1. (20 points) A ball with mass $m$, starting at rest, is dropped from a height of $h_i$ and bounces on a hard floor. The force on the ball from the floor is shown in the figure. Find the height $h_f$ to which the ball rebounds. $\tau$ is an amount of time.
2. (30 points) You do a collision experiment with carts in the lab, but this time you work with expensive equipment that reduces friction with the track to a negligible level. You also work with carts that incorporate a spring that can be compressed and released during a collision, imparting the energy stored in the spring to the carts rebounding from the collision.

You set up the collision with a cart with mass $2m$ with initial velocity $v_{2i} = v$ heading toward a cart with mass $m$ that starts at rest. You measure the final velocity of the cart with mass $m$ in three different experiments, obtaining $v_{1f} = v$, $v_{1f} = \frac{4}{3}v$, and $v_{1f} = 2v$. Analyze these three experiments and determine which experiments must have had a compressed spring released during the collision.
3. (50 points) Remember how we got the gravitational potential energy \( mgh \): the applied force acting against gravity had a magnitude of \( mg \), and we found the area under the force-versus-distance curve, a rectangle of height \( mg \) and base \( h \).

Now we want to generalize this to beyond locations close to the Earth’s surface. Take the gravitational force magnitude \( F_G \) between two point masses \( m_1 \) and \( m_2 \) separated by a distance \( r \). We will again look at the area under the force-distance curve.

(a) Sketch a graph of \( F_G \) versus \( r \).

Now, according to your sketch, do you do more work in changing \( r \) from \( R \) to \( 1.1R \), from \( 2R \) to \( 2.1R \), or from \( 3R \) to \( 3.1R \)?

(b) The convention for gravitational potential energy is to say that it is zero when the masses are infinitely far from each other. So the expression for \( U_G \) must become very small as \( r \) becomes large. Given this, and the behavior you found in part (a), which of the following is the correct general equation for \( U_G \)? (Only one of the options given is consistent with what you found about \( U_G \).)

(i) \( U_G = \frac{1}{2} Gr^2 \)
(ii) \( U_G = m_1 m_2 r \)
(iii) \( U_G = \frac{m_1}{m_2} e^{-Gr} \)
(iv) \( U_G = - \ln Gr \)
(v) \( U_G = -Gm_1 m_2 / r \)

Show your work checking consistency:
(c) Given your $U_G$, find the escape speed of an object launched away from Earth. This is the minimum speed necessary to never fall back to Earth under the influence of gravity: You start from $r$ equal to the radius of Earth and speed equal to your escape speed, and end up at $r$ equal to infinity and the object at rest. You can look up data about the Earth to find a numerical result.

(d) Find an equation for the radius $r_s$ for the event horizon of a black hole with mass $m$. The event horizon marks the point beyond which nothing can return, since it would have to travel faster than light. You find $r_s$ by setting the escape speed equal to the speed of light $c$. 