

---

## Homework Solutions 3 (Schroeder Chapter 3)

### 10

- (a) Melting (at constant temperature) increases entropy by

$$\Delta S_a = \frac{Q}{T_i} = \frac{mL}{T_i} = 36.6 \text{ J/K}$$

- (b) Integrate for the entropy due the temperature increase:

$$\Delta S_b = C \int_{T_i}^{T_f} \frac{dT}{T} = C \ln \frac{T_f}{T_i} = 11.0 \text{ J/K}$$

- (c) The kitchen is at a constant temperature, and the heat transferred to the ice and then water is lost by the kitchen. So the kitchen entropy changes by

$$\Delta S_c = -\frac{mL + mC(T_f - T_i)}{T_f} = -44.1 \text{ J/K}$$

- (d) Add all these to get the change in entropy for the universe:

$$\Delta S_{\text{total}} = \Delta S_a + \Delta S_b + \Delta S_c = 3.5 \text{ J/K}$$

This is positive, as it should be.

### 25

- (a) Using  $\Omega$ ,

$$S = k \ln \Omega = k \left( q \ln \frac{q+N}{q} + N \ln \frac{q+N}{N} \right)$$

- (b) Taking the derivative,

$$\begin{aligned} \frac{1}{T} &= \frac{\partial S}{\partial U} = \frac{1}{\epsilon} \frac{\partial S}{\partial q} = \frac{k}{\epsilon} \frac{\partial}{\partial q} [(q+N) \ln(q+N) - q \ln q - N \ln N] \\ &= \frac{k}{\epsilon} \left[ \ln(q+N) + \frac{q}{q+N} - \ln q - \frac{q}{q} + \frac{N}{q+N} \right] = \frac{k}{\epsilon} \ln \left( 1 + \frac{N}{q} \right) \end{aligned}$$

Therefore

$$T = \frac{\epsilon}{k \ln(1 + N\epsilon/U)}$$

(c) Inverting,

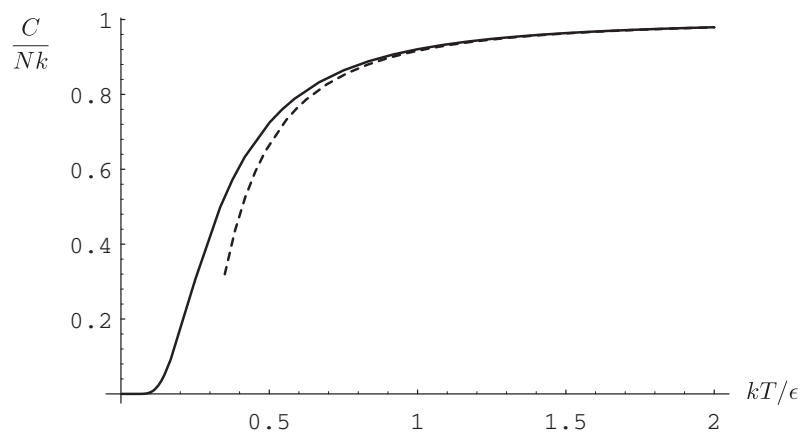
$$U = \frac{N\epsilon}{e^{\epsilon/kT} - 1}$$

$$C = \frac{\partial U}{\partial T} = \frac{N\epsilon^2}{kT^2} \frac{e^{\epsilon/kT}}{(e^{\epsilon/kT} - 1)^2}$$

(d) When  $T \rightarrow \infty$ ,  $kT \gg \epsilon$  and  $e^{\epsilon/kT} \approx 1 + \epsilon/kT$ . So we get

$$C \approx \frac{N\epsilon^2}{kT^2} \frac{1}{(\epsilon/kT)^2} = Nk$$

(e) The solid line is the exact result, the dashed line is the approximation in (f).



(f) With  $x = \epsilon/kT$ ,

$$\frac{C}{Nk} \approx \frac{x^2(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3)}{(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 - 1)^2}$$

$$\approx \frac{1 + x + \frac{1}{2}x^2}{1 + x + \frac{7}{12}x^2} \approx 1 - \frac{1}{12}x^2$$

## 34

(a) Since each link is oriented toward the right or the left, this is just like a chain of spins that are up or down. We've done that;

$$S = k \ln \left( \frac{N}{N_R} \right) \approx k [N \ln N - N_R \ln N_R - (N - N_R) \ln(N - N_R)]$$

---

(b)  $L = l(N_R - N_L) = l(2N_R - N)$ .

(c)  $dU = T dS + F dL$ .

(d) The identity gives

$$F = -T \left( \frac{\partial S}{\partial L} \right)_U \Rightarrow F = -T \frac{\partial S}{\partial N_R} \frac{\partial N_R}{\partial L} = -\frac{\partial S}{\partial N_R} \frac{T}{2l}$$

Using the expressions for  $S$  and  $L$ ,

$$F = -\frac{kT}{2l} \ln \left( \frac{N - N_R}{N_R} \right) = -\frac{kT}{2l} \ln \left( \frac{2}{L/Nl + 1} - 1 \right)$$

(e) With  $L \ll Nl$ ,

$$\frac{2}{L/Nl + 1} - 1 \approx 1 + \frac{2L}{Nl} \Rightarrow F \approx \frac{kT}{Nl^2} L$$

(f)  $F \propto T$ . At high temperatures, the links are more randomly oriented, so the chain will tend to shorten.

(g) Adiabatic means no entropy change. A decrease in the  $S$  we calculated here, because the configuration becomes less random, will have to be compensated by an increase in the entropy due to the vibration: the temperature will increase.

### 37

(a) The energy of the gas is  $U = U_0 + Nmgz$ , where  $U_0$  is for an ideal gas at  $z = 0$ . So

$$\mu(z) = \left( \frac{\partial U}{\partial N} \right)_{S,V} = \mu_0 + mgz$$

But  $\mu_0 = \partial U_0 / \partial N$  is given in the textbook, equation (3.63).

(b) Equilibrium means  $\mu_z = \mu_0$ . Therefore

$$-kT \ln \left[ \frac{V}{N(z)} \left( \frac{2\pi mkT}{h^2} \right)^{3/2} \right] + mgz = -kT \ln \left[ \frac{V}{N(0)} \left( \frac{2\pi mkT}{h^2} \right)^{3/2} \right]$$

Canceling out the terms that appear in both logarithms,

$$kT \ln N(z) + mgz = kT \ln N(0) \Rightarrow N(z) = N(0) e^{-mgz/kT}$$