1. (20 points) In the physics lab, you notice that a spring you’re playing around with has a period of exactly 2.00 seconds when you attach a mass hanger and three masses, each with a mass of 50.0 g, and extend the spring down by exactly 11.5 cm and let it go. You then think it would be amusing if you were to use the spring as an egg timer, and borrow the spring from me and take it home.

Next morning, you prepare to boil a three-minute egg. But you realize that you’ve forgot to borrow a ruler from the lab, and you have no way to measure the spring extension. Moreover, you brought the 50.0 g mass holder with you, but none of the masses. You’re caught in a snow storm and there’s no way to come down to campus and collect the equipment you need. Worse, you have nothing to keep time with where you live—your cell phone, for example, has a dead battery.

How, then, would you use the spring to time your three-minute egg? Describe exactly what you would do, using the appropriate equations when needed for your reasoning.

Answer: You check your physics notes and find that the period of a spring is \( T = \frac{2\pi}{\omega} = 2\pi \sqrt{m/k} \). This doesn’t depend on the amplitude, so the extension of the spring when you release it is irrelevant. You don’t need a ruler.

You then also notice that \( T \propto \sqrt{m} \). If you use only the mass hanger, you will be oscillating the same spring with 1/4 of the original mass attached. Therefore, your new period of oscillation will be \( \sqrt{1/4} = 1/2 \) times the original period.

You start the mass hanger bouncing around, confident that each period will now be 2.00/2 = 1.00 s. Three minutes is 180 seconds, so you just need to count to 180 to get your egg just right.

2. (30 points) You have the following interferometer setup in a lab, with a wave source labeled S, detector D, and a mirror to produce two wave paths between source and detector. There is a distance of \( d \) between the source and the mirror, and you keep the length \( h \) constant while changing \( d \). The wavelength of the waves is \( \lambda \).
Make a qualitative graph of how the intensity detected will depend on the distance $d$. Be careful about the relative peak heights, and whether the intensity is ever exactly zero. Explain your decisions, and how you made your graph.

Answer: There are two paths for waves between the source and detector. You can see from the geometry of the experiment that the path length difference is $l_1 - l_2 = d$. There is also a phase shift of $\lambda/2$ due to the reflection off the mirror. Therefore, the condition for constructive interference is $d = (m + \frac{1}{2})\lambda$, while destructive interference happens at $d = m\lambda$, where $m$ is an integer.

The intensity of the waves will decrease with distance from the source. Therefore, the peak heights will become smaller as $d$ increases. And since the wave amplitudes will, for the same reason, not match exactly, there will be no full cancellation and no $I = 0$ except for $d = 0$ when the source and mirror are practically on top of each other.

3. (30 points) You have charges $-q$ at $x = 0, y = a$ and $x = 0, y = -a$. The positions of the charges are fixed.
(a) Find the total electric field at a point a distance \(x\) from the origin on the \(x\) axis. In other words, find the total \(E_x\) and \(E_y\) as functions of \(k\), \(q\), \(a\), and \(x\).

**Answer:** For the electric fields due to the individual charges we have, with the top charge 1 and the bottom number 2,

\[
E_{1x} = -E_1 \cos \theta \quad E_{1y} = E_1 \sin \theta
\]

\[
E_{2x} = -E_2 \cos \theta \quad E_{2y} = -E_2 \sin \theta
\]

where, as with examples you’re familiar with, \(E_1 = E_2 = \frac{kq}{r^2}\), \(\cos \theta = \frac{x}{r}\), \(\sin \theta = \frac{a}{r}\), and \(r = \sqrt{x^2 + a^2}\). With these in place,

\[
E_{1x} = -kq \frac{x}{(x^2 + a^2)^{3/2}} \quad E_{1y} = kq \frac{a}{(x^2 + a^2)^{3/2}}
\]

\[
E_{2x} = -kq \frac{x}{(x^2 + a^2)^{3/2}} \quad E_{1y} = -kq \frac{a}{(x^2 + a^2)^{3/2}}
\]

Therefore, for the total electric field \(\vec{E}\) we get

\[
E_x = E_{1x} + E_{2x} = -2kq \frac{x}{(x^2 + a^2)^{3/2}}
\]

\[
E_y = 0
\]

This is toward the left.
(b) When \( x \gg a \) (\( x \) is much larger than \( a \)), \( x + a \approx x, \ x^2 + a^2 \approx x^2 \), and so on. Using such approximations, find out how the electric field behaves very far from the two charges. (Hint: This should be a mathematically simple expression.)

**Answer:** Using \( x^2 + a^2 \approx x^2 \),

\[
E_x \approx -2kq \frac{x}{(x^2)^{3/2}} = -2kq \frac{x}{x^3} = k\left(-\frac{2q}{x^2}\right)
\]

(c) Interpret your result for the electric field far away. Physically, *what does it mean?* Is it what you would expect?

**Answer:** Notice that your expression is the same as what you would get for a charge of \(-2q\) at \( x = 0 \) and \( y = 0 \). And \(-2q\) is the total charge we have on hand. Far away from a charge distribution, it behaves like a single charge equal to the total charge. That shouldn’t be surprising.

4. **(20 points)** First, draw equipotential lines and electric field lines for a dipole-like arrangement you investigated in the lab. The low-voltage point in the water is at \(-3\) V, and the high-voltage point is at \(+3\) V. Draw the equipotential lines at 1 V intervals.
Then, draw equipotential lines at 1 V intervals and electric field lines for the case where you have a point at +3 V and a plate at 0 V. Only draw what is happening on the left side; you can ignore the right side of the plate. Also draw some appropriate charges on the plate to give a qualitative idea of the charge distribution on the plate. Explain your reasoning.

How are your two graphs related—how did drawing the dipole first help you draw the second graph with the plate?

**Answer:** The conducting plate is itself an equipotential. You will notice that the given equipotential lines at 3 V and 0 V are the same for the dipole and the charge-and-plate arrangements. Indeed, *everything* on the left side will be the same for the two arrangements, because the given voltage constraints are identical.

The charges on the plate will be negative—it’s the low voltage end. They won’t be distributed equally throughout the plate, since they are attracted toward the + charge.