

Solutions to Assignment 6; Phys 186

1. (40 points) An important discovery in the 1980s was the W^+ and W^- , which are among the particles responsible for the weak nuclear force. W^+ and W^- are antiparticles of each other, and they each have a rest mass of $80.4 \text{ GeV}/c^2$. Say you want to create a W^+ and W^- pair by a head-on collision of an electron and a positron (e^- and e^+) into each other at speeds close to the speed of light. The rest mass of an electron (and a positron, its antiparticle) is $0.511 \text{ MeV}/c^2$. (Remember: $1 \text{ GeV} = 1000 \text{ MeV}$.)

- (a) As observed from the lab frame of reference, the e^- and e^+ head toward each other with equal and opposite velocities in the collision. What is the minimum time dilation factor γ that the e^- and e^+ must have in order to produce enough energy to create a W^+ and W^- pair at rest?

Answer: Energy conservation means that

$$2m_W c^2 = 2\gamma m_e c^2 \quad \Rightarrow \quad \gamma = \frac{m_W}{m_e} = 1.57 \times 10^5$$

This is an enormous γ ; the speed of the electron must be almost c .

- (b) Say that in the lab frame of reference, the electron traveled 30.0 km at a constant speed corresponding to the γ you calculated in (a). How far did it travel in its own frame of reference?

Answer: In the lab frame, the 30.0 km distance is at rest; therefore, the proper length $\Delta x_0 = 30.0 \text{ km}$. This distance in the electron's frame is contracted, as it is moving: the endpoint of the electron's flight is rushing at very high speed toward the stationary electron. Length contraction: $\Delta x = \Delta x_0/\gamma$, therefore

$$\Delta x = \frac{3 \times 10^4 \text{ m}}{1.57 \times 10^5} = 0.191 \text{ m}$$

- (c) Calculate how long it took for the electron to travel 30.0 km in the lab frame of reference. Then calculate how long this time interval was in the electrons own frame of reference.

Answer: The speed is almost c . Therefore, to travel 30 km,

$$\Delta t = \frac{\Delta x_0}{c} = \frac{3 \times 10^4 \text{ m}}{3 \times 10^8 \text{ m/s}} = 10^{-4} \text{ s}$$

Note that this is *not* the proper time: in the lab frame, the beginning and end of the electron's flight do not take place at the same location. In the electron's frame, however, since the endpoint is rushing toward a stationary electron, the beginning and ending events happen at the same location. So the electron will have the proper time,

$$\Delta t_0 = \frac{\Delta t}{\gamma} = \frac{\Delta x}{c} = 6.37 \times 10^{-10} \text{ s}$$

2. (30 points) A cosmic ray collision creates a muon (a subatomic particle) near the top of the troposphere, at an altitude of 9000 m. The muon heads straight towards the surface at a speed of $0.998c$.

- (a) In the reference frame of a ground observer, what is the muon's initial distance to the surface? What is the time the muon takes to reach the surface?

Answer: In this reference frame you measure Δx_0 , which is 9000 m. The time to the surface is $(9000 \text{ m})/(0.998c) = 3.01 \times 10^{-5} \text{ s}$. There is no relativity involved, since we do not switch between frames of reference: the question entirely concerns the ground frame of reference. So no γ factors are involved.

This time interval is Δt , a dilated time. Since in the ground frame, the creation of the muon and its reaching the surface take place at different locations (9000 m high and 0 m high), this is not the proper time. However, since the troposphere and the surface are stationary in the ground frame, the proper length $\Delta x = 9000 \text{ m}$ is in this frame.

- (b) In the reference frame of the muon, what is the muon's initial distance to the surface? What is the time the muon takes to reach the surface?

Answer: In the muon's frame of reference, the ground is rushing up to meet the muon: the beginning and ending locations are not stationary. Therefore the length (height) in the muon's frame will be contracted: $\Delta x = \Delta x_0/\gamma$. The time dilation factor $\gamma = 1/\sqrt{1 - 0.998^2} = 15.8$. So $\Delta x = 569$ m. In this reference frame the ground is rushing up at $v = 0.998c$. The time it takes to meet it is $\Delta x/v = 1.9 \times 10^{-6}$ s. This is Δt_0 , since the beginning and end events (muon being created and muon reaching the ground) happen at the same location: exactly where the muon is.

- (c) When measured at rest in the lab, the average lifetime of a muon is 2.2×10^{-6} s. Given your answers to (a) and (b), would an average muon make it to the surface, or does it have to be an exceptionally long-lived one? Explain.

Answer: The muon's frame of reference is where the muon is stationary, as in the lab experiments that established the average lifetime. Therefore we should compare Δt_0 we just calculated to the lifetime. Since $1.9 \times 10^{-6} < 2.2 \times 10^{-6}$, the average muon has enough time to make it to the surface.

3. (30 points) Special relativity tells you that time and space are weird. But are they so weird as to allow the following?

- (a) Lengths contract in frames of reference that are moving relative to the length in question. Is there a frame of reference where the measured length would become negative? *Hint:* Ask yourself: What sort of γ and v for a frame of reference would allow this? Is this possible?

Answer: With length contraction, $\Delta x = \Delta x_0/\gamma$. Proper lengths are non-negative: $\Delta x_0 \geq 0$. Since $\gamma = 1/\sqrt{1 - (v/c)^2}$ cannot be negative, you can contract lengths down to zero if you were to travel at exactly $v = c$, but you cannot go negative.

- (b) A pilot of a spaceship traveling close to the speed of light can age slower than those who remain behind, and can therefore time travel into the future. Can she also time travel into the past? *Hint:* Ask yourself: what sort of Δt_0 would the pilot need to travel into the past? What sort of γ and v for a frame of reference would allow this? Is this possible?

Answer: With time dilation, you have $\Delta t = \gamma \Delta t_0$. The proper time $\Delta t_0 > 0$. To travel into the future, you need $\Delta t > \Delta t_0 > 0$, since that way, less time passes for you relative to the events you're interested in. Since for all speeds v , $1 < \gamma < \infty$, this is not a problem. So you can travel into the future.

But for time travel into the past, you need a negative sign: $\Delta t < 0$. You can't get that with any speed v . Sometimes in science fiction, you hear that if you could go faster than the speed of light, you'd travel back in time. If you try to calculate γ for $v > c$ you will attempt to take the square root of a negative number.