

Solutions to Assignment 7; Phys 186

1. (20 points) A physics major tells you that they have a new explanation of black holes. They ask you to imagine a brick, which we heat up to higher and higher temperatures.

- As temperature increases, the atoms making up the brick will jiggle more vigorously.
- Atoms are made of electrically charged particles.
- The atoms will radiate higher amplitude electromagnetic waves.
- More electromagnetic waves in the brick leads to increased destructive interference between the waves;
- Therefore the intensity of the radiation escaping from the brick will decrease.
- Beyond a threshold, *no* radiation will escape: the brick will be completely black.
- We can calculate the threshold using $E = mc^2$. Since mass is equivalent to energy, heating up the brick is equivalent to adding more and more mass.

This reasoning is incorrect. Explain how it goes wrong. Circle the suspicious bullet points.

Answer: Things start falling apart with the “increased destructive interference” point. No such thing happens; thermal motion is random jiggling, so the radiation due to thermal motion is a chaotic mess of different phases and wavelengths that cannot produce any coherent interference of any sort. In fact, there will be an increasing intensity of radiation, proportional to T^4 as we learned in the first semester. The rest is blather; while adding more energy *is* equivalent to adding more mass, the brick will be vaporized long before any measurable increase in mass will take place. That’s because the energy that you need to heat up a brick is much much smaller than mc^2 for a brick.

2. (30 points) For the following, you will need the expression for the radius of the event horizon of a black hole that we derived.

- (a) One way to get a black hole is to squeeze lots of mass into a very small volume. Almost all the mass of a Hydrogen atom is squeezed into the very small volume of a proton. Are protons dense enough to be black holes? Answer this question by using reasonable numbers about protons you can look up. Write down the sources of your numbers.

Answer: The event horizon of a black hole has radius $r = 2Gm/c^2$. For the proton mass of $m = 1.67 \times 10^{-27}$ kg, we get $r = 2.48 \times 10^{-54}$ m. The radius of the proton is around 10^{-15} m, since that is the order of magnitude for nuclear distances. Whatever the exact value you take, clearly the proton radius is *much, much* larger than what it would need to be for a black hole.

- (b) You stand such that your lower legs, with mass 5.0 kg, is at the event horizon of a 30 solar mass black hole, and your 5.0 kg head is just 1.0 m further away from the horizon. Find the difference between the gravitational forces felt by your head and your lower legs. What effect on you would this difference in forces have? (Math hint: When $d \ll r$, $\frac{1}{r^2} - \frac{1}{(r+d)^2} \approx \frac{2d}{r^3}$.)

Answer: The difference between gravitational forces is

$$\Delta F = GMm \left(\frac{1}{r^2} - \frac{1}{(r+d)^2} \right) \approx \frac{2GMmd}{r^3}$$

where $m = 5$ kg, $M = 30 \times 1.99 \times 10^{30}$ kg, $d = 1$ m, and $r = 2GM/c^2 = 8.85 \times 10^4$ m. With all the numbers in, we get $\Delta F = 5.7 \times 10^7$ N. This is very large—you will be ripped apart.

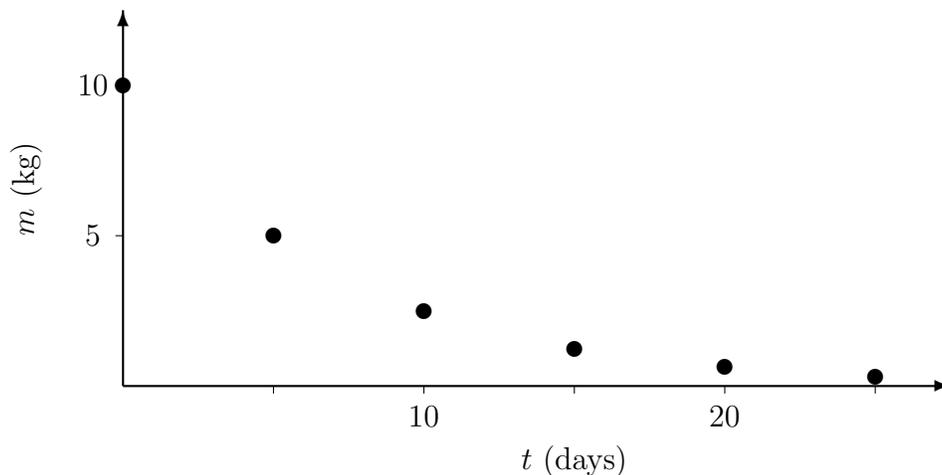
3. (30 points) Let's do half-lives.

- (a) You have friend who is not a science major. She tells you that quantum mechanical events cannot be truly random. After all, randomness implies unpredictability, but physicists make precise predictions using quantum mechanics. Given the half-life, they can tell you exactly what

amount of a radioactive sample will remain after a certain time. Given the energy of photons emitted from a light source, they can calculate the interference pattern observed when a diffraction grating is placed between the source and a screen. Correct your friends' misconceptions and explain what the role of randomness in quantum mechanics is.

Answer: *Individual* quantum events are random. We cannot predict when an individual nucleus will decay, or the exact path any single photon will take. But precisely *because* individual events are random, if lots and lots of individual random events take place, a very reliable statistical pattern will emerge. We get very definite half-lives and interference patterns when the number of events is about in the 10^{20} 's.

- (b) You have a 10.0 kg block of radioactive material A , and at time $t = 0$, you start with all 10 kg being pure A . Sketch a graph of the amount of A that remains in the block over time. The half-life of A nuclei is 5.0 days. **Answer:**



(And connect the dots.)

- (c) Pick, from among the following, the correct expression for the amount of A remaining over time. Here $\tau = t_{1/2}/\ln 2$, and $m_0 = 10$ kg.

(a) $m = m_0 \cos \frac{t}{\tau}$

$$(b) \quad m = m_0 \left(1 - \frac{t}{\tau}\right)$$

$$(c) \quad m = m_0 \ln \frac{t}{\tau}$$

$$(d) \quad m = m_0 e^{-\frac{t}{\tau}}$$

$$(e) \quad m = \frac{m_0}{\sqrt{1 - \left(\frac{t}{\tau}\right)^2}}$$

(d) What is m at $t = 9.0$ days?

Answer: $m = 10 e^{-(9 \ln 2)/5} = 2.9 \text{ kg}$

4. (20 points) There have been occasional concerns that people living close to electric power lines might have high risks of cancer due to their proximity to electromagnetic radiation emitted by the power lines. Determine, using a simple calculation, whether this concern is warranted. I will supply any physical data that you need—just ask me. I'm interested in whether you know to ask the right questions about the numbers that you might need.

Answer: Power lines carry alternating current, with a frequency of 60 Hz. That means accelerating electric charges with a 60 Hz frequency, producing predominantly 60 Hz electromagnetic waves. Photons with this frequency have an energy of

$$E = hf = 4.0 \times 10^{-32} \text{ J} = 2.5 \times 10^{-13} \text{ eV}$$

Chemical bond energies are of the order of eV's. So the photons produced by power lines have energies that are extremely smaller than bond energies. Therefore they cannot disrupt biologically important molecules such as DNA, and therefore it is hard to see any physical mechanism for their causing cancer.