Homework Solutions 6 (Ryden Chapter 8 & 9)

8.4 The distance to the surface of last scattering is almost the horizon distance, but not quite. So

$$d_p = c\int_{t_{\rm ls}}^{t_0} \frac{dt}{a} = c\int_0^{t_0} \frac{dt}{a} - c\int_0^{t_{\rm ls}} \frac{dt}{a} = d_{\rm hor} - c\int_0^{t_{\rm ls}} \frac{dt}{a}$$

This last term comes from the very early, radiation and matter dominated universe. Looking this up from chapter 5, we find equation 5.109 is applicable. Therefore

$$\int_0^{t_{\rm ls}} \frac{dt}{a} = \frac{1}{H_0\sqrt{\Omega_{r,0}}}\int_0^{a_{\rm ls}} \frac{da}{\sqrt{1+a/a_{\rm rm}}} = \frac{2a_{\rm rm}}{H_0\sqrt{\Omega_{r,0}}}\left(\sqrt{1+\frac{a_{\rm ls}}{a_{\rm rm}}}-1\right)$$

Some of the values we need are in table 5.2. From our basic $a(t_e) = 1/(1+z)$, we also have $a_{\rm ls} = 1/(1+z_{\rm ls}) = 1/1091$.

The horizon distance in the Benchmark model is $3.20\,c/H_0$. Putting this in, we get

$$d_p = \frac{3.20\,c}{H_0} - \frac{0.064\,c}{H_0} = \frac{3.14\,c}{H_0} = 1.37\times 10^{10}\,\text{pc}$$

The Benchmark model has $\kappa = 0$, so d_L is not complicated:

$$d_L = (1+z_{\rm ls})d_p = 1.50\times 10^{13}\,\text{pc}$$

9.2 This one is fairly trivial. The result in equation 9.17 in the book holds exactly, but just with a different Q_n value.

$$\frac{n_n}{n_p} \approx \exp\left(-\frac{0.129}{0.8}\right) = 0.85$$

Since now $Q_n < m_e c^2$, neutrons cannot β-decay, and the temperature will be too low for significant neutron decay by capturing e^+—the positrons will be gone.

Again, following the textbook, if all the neutrons go into He, this ratio means each He will get 2 n and 2 p with 0.35 p left over. Following the reasoning of equation 9.21, the mass fraction of He will be

$$Y_{\rm max} = \frac{4}{4.35} = 0.92$$

9.3 The total energy output by our galaxy is approximately its luminosity times its age, which, with the appropriate unit conversions, is 2.3×10^{73} eV. Dividing that by the energy released by the creation of each He, we get that the number of He is 8×10^{65}.

Again, doing the requisite annoying unit conversions and so on, the ratio of the total He mass produced to the total baryon mass is 0.027. This is the *increase* over the primordial value of 0.24.