Homework Solutions 6 (Ryden Chapter 8 & 9)

8.4 The distance to the surface of last scattering is almost the horizon distance, but not quite. So

\[ d_p = c \int_{t_{ls}}^{t_o} \frac{dt}{a} = c \int_{0}^{t_{ls}} \frac{dt}{a} - c \int_{0}^{t_{ls}} \frac{dt}{a} = d_{hor} - c \int_{0}^{t_{ls}} \frac{dt}{a} \]

This last term comes from the very early, radiation and matter dominated universe. Looking this up from chapter 5, we find equation 5.109 is applicable. Therefore

\[ \int_{0}^{t_{ls}} \frac{dt}{a} = \frac{1}{H_0 \sqrt{\Omega_r,0}} \int_{0}^{a_{ls}} \frac{da}{\sqrt{1 + a/a_{rm}}} = \frac{2a_{rm}}{H_0 \sqrt{\Omega_r,0}} \left( \sqrt{1 + \frac{a_{ls}}{a_{rm}}} - 1 \right) \]

Some of the values we need are in table 5.2. From our basic \( a(t_e) = 1/(1 + z) \), we also have \( a_{ls} = 1/(1 + z_{ls}) = 1/1091 \).

The horizon distance in the Benchmark model is \( 3.20 c/H_0 \). Putting this in, we get

\[ d_p = 3.20 \frac{c}{H_0} - \frac{0.064 c}{H_0} = 3.14 \frac{c}{H_0} = 1.37 \times 10^{10} \text{ pc} \]

The Benchmark model has \( \kappa = 0 \), so \( d_L \) is not complicated:

\[ d_L = (1 + z_{ls})d_p = 1.50 \times 10^{13} \text{ pc} \]

9.2 This one is fairly trivial. The result in equation 9.17 in the book holds exactly, but just with a different \( Q_n \) value.

\[ \frac{n_n}{n_p} \approx \exp \left( -\frac{0.129}{0.8} \right) = 0.85 \]

Since now \( Q_n < m_e c^2 \), neutrons cannot \( \beta \)-decay, and the temperature will be too low for significant neutron decay by capturing \( e^+ \)—the positrons will be gone.

Again, following the textbook, if all the neutrons go into He, this ratio means each He will get 2 \( n \) and 2 \( p \) with 0.35 \( p \) left over. Following the reasoning of equation 9.21, the mass fraction of He will be

\[ Y_{\text{max}} = \frac{4}{4.35} = 0.92 \]
The total energy output by our galaxy is approximately its luminosity times its age, which, with the appropriate unit conversions, is $2.3 \times 10^{73}$ eV. Dividing that by the energy released by the creation of each He, we get that the number of He is $8 \times 10^{65}$.

Again, doing the requisite annoying unit conversions and so on, the ratio of the total He mass produced to the total baryon mass is 0.027. This is the increase over the primordial value of 0.24.