College Physics I
Lab 8: Cooling

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Relationships

Many mathematical relationships arise in physics. For example, you know that for constant acceleration, the velocity v depends linearly on time Δt: $v_f = v_i + a\Delta t$ where v_i and a are constants. Thus, if you graphed v versus Δt for an object undergoing constant acceleration, you would expect a straight line. Likewise, for constant acceleration starting from rest, the position x is proportional to the square of the time: $x_f = x_i + \frac{1}{2}a(\Delta t)^2$, where a is a constant. If you graphed x versus Δt for an object undergoing constant acceleration, you'd expect to see a curve—a parabola. If you wanted to see a straight line, you would have to graph x versus $(\Delta t)^2$.

A relationship you may not be familiar with, but which is actually more important in describing our world than the two examples above, is when one variable depends on the number e raised to the power of the other variable. If f is the dependent variable and x is the independent variable, then such a relationship would look like this:

$$f = f_0\, e^{\frac{x}{x_0}},$$

where f_0 and x_0 are just constants inserted to make sure the units on the left match up with the units on the right. The number e is unitless, and whatever is in the exponent must also be unitless.

Such a relationship is called an *exponential relationship*. Exponential relationships occur whenever the rate at which something changes depends on however much of that something there is. For example, if you want to make simple model describing the amount of bacteria there is in a petri dish at a particular time, the amount would depend on time exponentially, since the rate at which new bacteria appear is proportional to how many there already are.

You may have noticed that when you take a hot drink outside on a cold day, it cools down rapidly at first, and then more slowly as the temperature of the drink gets closer and closer to the temperature of the surroundings. This is characteristic of an exponential relationship—the rate at which the temperature of the drink changes at a particular moment depends on the current temperature difference between the drink and the surroundings at that particular moment. Let ΔT be the temperature difference between the drink and the surroundings: $\Delta T = T_{\mathsf{drink}} - T_{\mathsf{surroundings}}$. Then a simple model for cooling might be:

$$\Delta T = \Delta T_0\, e^{-\frac{t}{\tau}}$$

where ΔT_0 is the temperature difference at time $t = 0$, and τ is a constant with units of time. Assuming our model is correct, if you graph ΔT versus t, you will not get a straight line. There is, on the other hand, something you could graph in this situation which would give a straight line:

$$\begin{aligned}
\Delta T &= \Delta T_0\, e^{-\frac{t}{\tau}} \\
\frac{\Delta T}{\Delta T_0} &= e^{-\frac{t}{\tau}} \\
\ln\left(\frac{\Delta T}{\Delta T_0}\right) &= -\frac{t}{\tau} \\
\ln \Delta T - \ln \Delta T_0 &= -\frac{t}{\tau} \\
\ln \Delta T &= \ln \Delta T_0 - \frac{t}{\tau}
\end{aligned}$$

This means that if you graph the natural log of the temperature difference versus time, $\ln \Delta T$ versus t, you should get a straight line, the slope of which is equal to $-1/\tau$. Furthermore, this line should cross the "$\ln \Delta T = 0$" axis at the value $\ln \Delta T_0$. Here, τ is a time constant which has to do with the properties of the object that is cooling, and of the interface between that object and the surroundings. Let's call τ the time constant for the system. If τ is very big, then it will take a long time for the object to cool. If τ is very small, then the object will cool very quickly. But in both cases the object will cool exponentially!

Activity 1: Make sure you understand

To make sure you understand the model described above, answer the following questions before you take any data. Suppose that at time $t = 0$, the temperature of the object was 100°C. Furthermore, suppose that the temperature of the surroundings was 20°C:

1. What is ΔT_0?
2. As time passes, will ΔT increase, decrease, or stay the same?
3. After a very, very long time, what will ΔT be, for all intents and purposes?
4. Just to get a feel for what an exponential function looks like, make qualitative graphs of the functions $f_1 = e^x$ and $f_2 = e^{-x}$.
5. Based on your graphs from question 4, what is f_1 when $x \to \infty$? What is f_2 when $x \to \infty$? Would f_1 or f_2 best describe population growth, assuming there was no war, famine, sickness, birth control, etc.?

To hand in for activity 1

Answers to all questions.

Activity 2: Heat the object

Check the room temperature in the region where you are going to let your object cool. Call it $T_{\text{surroundings}}$ and write it down.

Take an aluminum cylinder with a hole in it, put a thermometer into the hole, and place it on a hot plate. Turn the hot plate up to high, and wait for the temperature to go just above 100°C. Then remove the cylinder from the hot plate, turn off the hot plate and unplug it. Wait for about two minutes to give the thermometer time to adjust.

To hand in for activity 2

Nothing.

Activity 3: Monitor the temperature of the object as it cools

The temperature, T_{object}, should start dropping. After it has started dropping, set the stopwatch at zero, and at that moment note the temperature of the thermometer. Henceforth, note time on the stopwatch every time the thermometer reads an exact degree, such as 93.0°C, 92.0°C, and so forth. Do this for approximately twenty-five minutes.

To hand in for activity 3

All measurements: $T_{\text{surroundings}}$, and T_{object} and t for each data point.

Activity 4: Calculate derived quantities from your data

Find ΔT_0. For each data point, find ΔT.

To hand in for activity 4

All quantities calculated from the initial data. It would be best if you presented the values from activities 3 and 4 together in one easy to follow chart.

Activity 5: Graphical interpretation

Graph ΔT versus t from the data you collected. Is the shape of the graph what you expected? Graph $\ln \Delta T$ versus t. Assuming it is a straight line, find the slope of the best-fit line, and from that find τ for this system. Note that if Excel gives you only one or two significant figures on the slope, you will need to adjust the Trendline parameters to increase the number of digits.

Now, what does τ mean—what is its significance? To begin answering this question, start with another question: what are the units for τ?

To hand in for activity 4

Two graphs, result for τ, and your interpretation of what τ means.