Homework Solutions 1 (Schroeder Chapter 1)

19 Oxygen is O_2, which is about 16 times the mass of H_2. In equation (1.21) we have

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

Therefore the rms and the average speeds of O_2 will be smaller by a factor of $\sqrt{16} = 4$.

24 If nothing is frozen out, each Pb atom will have the six quadratic degrees of freedom typical of a solid: one kinetic and one potential energy in each of the three orthogonal directions. The atomic mass for Pb is about 207, and therefore $n \approx 1/207$ for a gram of Pb. So

$$U_{\text{thermal}} = f\,\frac{1}{2}NkT = f\,\frac{1}{2}nRT = 36 \text{ J}$$

33 Step A increases V at constant P. The area under the P-V curve is positive, and so $W_A = -\int dV\, P < 0$. Similarly, $W_B = 0$ and $W_C > 0$. The total work done on the gas in the cycle is the negative of the area of the cycle, so $W_{\text{total}} = W_A + W_B + W_C > 0$.

Since for an ideal gas $U = f\frac{1}{2}NkT = f\frac{1}{2}PV$, $\Delta U_A > 0$, $\Delta U_B > 0$, and $\Delta U_C < 0$. Since the gas ends up at the same state where it starts, $\Delta U_{\text{total}} = \Delta U_A + \Delta U_B + \Delta U_C = 0$.

For each step, $Q = \Delta U - W$. Therefore $Q_A > 0$, $Q_B > 0$, and $Q_C < 0$. $Q_{\text{total}} = -W_{\text{total}} < 0$.

This cycle has work done on the gas, and releases heat into the environment, like a heat pump.

44 For a monatomic gas, we expect $C_P = \frac{5}{2}R = 20.8$ J/K·mol. For the noble gasses listed in the table, this seems to hold.

For diatomic gases, $C_P = \frac{7}{2}R = 29.1$ J/K·mol. This is reasonably close for CO, N_2, H_2, and O_2, but off for Cl_2—vibrational degrees of freedom must be contributing.

Polyatomic gases should have $C_P = \frac{8}{2}R = 33.3$ J/K·mol. Gaseous H_2O is close, and NH_3 and CH_4 are not too far off, but larger molecules appear to have considerable energy going into vibrational modes.

Solids should have $C_V = \frac{6}{2}R = 24.9$ J/K·mol; C_P should be only slightly higher. Al, Cu, Fe, and Pb are close. C, both as graphite and as diamond, is too low—vibrations must remain frozen out. Large molecules get complicated.

Liquids are not easy to make predictions for.

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(a) $w = y^2 z$ and $w = x^2/z$.

(b) $w(x, y) = xy$, and $w(x, z) = x^2/z$, so

$$\left(\frac{\partial w}{\partial x}\right)_y = y \quad \neq \quad \left(\frac{\partial w}{\partial x}\right)_z = \frac{2x}{z}$$

(c) $w(y, z) = y^2 z$, so

$$\left(\frac{\partial w}{\partial y}\right)_x = x \quad \neq \quad \left(\frac{\partial w}{\partial y}\right)_z = 2yz = 2x$$

$$\left(\frac{\partial w}{\partial z}\right)_x = -\frac{x^2}{z^2} \quad \neq \quad \left(\frac{\partial w}{\partial z}\right)_y = y^2 = \frac{x^2}{z^2}$$

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(a) The center of mass is halfway in between the particles. Say the distance between is $2r$. Then the kinetic energy is (with a factor of 2 because of two particles) $K = 2\frac{1}{2}mv^2 = mv^2$. The gravitational potential energy is $V = -Gm^2/(2r)$. Now, the gravitational force keeps the particles in circular motion, so

$$m\frac{v^2}{r} = \frac{Gm^2}{(2r)^2} \qquad \Rightarrow \qquad K = mv^2 = \frac{Gm^2}{4r} = -\frac{1}{2}V$$

Therefore $V = -2K$.

(b) The total energy is $U = K + V = K - 2K = -K$. Adding energy, therefore, decreases K.

(c) If the star is modeled as an ideal gas, its kinetic energy will be $\frac{3}{2}NkT$, so that $U = -K = -\frac{3}{2}NkT$. The heat capacity is then

$$C = \frac{dU}{dT} = -\frac{3}{2}Nk$$

This is negative.

(d) The only relevant quantities for the star as a whole are M and R, and we can also use the universal gravitational constant G. The way to combine these with the right units and that makes physical sense is $-GM^2/R$.

(e) Using (a) and (d),

$$\frac{3}{2}NkT \approx -\frac{1}{2}\left(-\frac{GM^2}{R}\right) = \frac{GM^2}{2R} \qquad \Rightarrow \qquad T \approx \frac{GM^2}{3NkR}$$

To get N, divide M by the mass of a proton and multiply by 2—since electrons are about 1/2000 the mass of a proton, we can neglect their mass. We end up with

$$T \approx 4 \times 10^6 \text{ K}$$

This is not unreasonable—the temperature of the sun is not the same throughout, varying from about 10^4 K at the surface to about 10^7 K at the center.