Homework Solutions 2 (Schroeder Chapter 2)

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(a) The total energy in each solid determines the macrostate. There are q_A units of energy in A, where $q_A = 0, 1, \ldots, 20$, and $q_B = 20 - q_A$ in B. So there are 21 macrostates.

(b) For the combined system, $q = N = 20$,

$$\Omega(20, 20) = \binom{20 + 20 - 1}{20} = \frac{39!}{20!\,19!} = 6.89 \times 10^{10}$$

(c) If we fix $q_A = 20$ and $q_B = 0$,

$$\begin{aligned} \Omega &= \Omega_A \Omega_B = \Omega(10, 20)\Omega(10, 0) = \binom{20 + 10 - 1}{20} \binom{0 + 10 - 1}{0} \\ &= \frac{29!}{20!\,9!} \frac{9!}{0!\,9!} = 1.00 \times 10^7 \end{aligned}$$

(d) With $q_A = q_B = 10$,

$$\begin{aligned} \Omega &= \Omega_A \Omega_B = \Omega(10, 10)\Omega(10, 10) = \binom{10 + 10 - 1}{10} \binom{10 + 10 - 1}{10} \\ &= \left(\frac{19!}{10!\,9!} \right)^2 = 8.53 \times 10^9 \end{aligned}$$

The equilibrium probability is

$$P = \frac{\Omega(10, 10)\Omega(10, 10)}{\Omega(20, 20)} = 0.124$$

(e) The Ω in (d) is about 10^3 times that in (c). So the system would tend to evolve toward (d) is if starts in (c), and the reverse evolution would be unlikely.

16 The number of microstates is 2^{1000}.

(a) An exactly even distribution has multiplicity

$$\Omega(500) = \binom{1000}{500} = \frac{1000!}{(500!)^2} \approx \frac{1000^{1000} e^{-1000} \left(2\pi \cdot 1000\right)^{\frac{1}{2}}}{\left[500^{500} e^{-500} \left(2\pi \cdot 500\right)^{\frac{1}{2}}\right]^2} = \frac{2^{1000}}{\left(500\pi\right)^{\frac{1}{2}}}$$

If every microstate is equally probable, the probability is then

$$P(500) = \frac{\Omega(500)}{2^{1000}} = \frac{1}{\left(500\pi\right)^{\frac{1}{2}}} = 0.025$$

(b) 600 heads means

$$\begin{aligned} \Omega(600) &= \binom{1000}{600} = \frac{1000!}{600!\,400!} \\ &\approx \frac{1000^{1000} e^{-1000} \left(2\pi \cdot 1000\right)^{\frac{1}{2}}}{600^{600} e^{-600} \left(2\pi \cdot 600\right)^{\frac{1}{2}} 400^{400} e^{-400} \left(2\pi \cdot 400\right)^{\frac{1}{2}}} \\ &= \frac{1000^{1000}}{600^{600} 400^{400} \left(480\pi\right)^{\frac{1}{2}}} \end{aligned}$$

$$P(600) = \frac{\Omega(600)}{2^{1000}} = \frac{500^{1000}}{600^{600} 400^{400} \left(480\pi\right)^{\frac{1}{2}}} = 4.6 \times 10^{-11} \ll P(500)$$

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(a) If $N/2$ point up,

$$\Omega = \binom{N}{N/2} = \frac{N!}{(N/2)!^2} \approx \frac{N^N e^{-N} \left(2\pi N\right)^{\frac{1}{2}}}{\left[(N/2)^{N/2} e^{-N/2} \left(\pi N\right)^{\frac{1}{2}}\right]^2} = 2^{N+1/2} (\pi N)^{-\frac{1}{2}}$$

For $N = 10^{23}$, $\Omega \approx 2^{10^{23}}$; huge.

(b) There are about 3×10^7 seconds in a year, so over 10^{10} years and 10^9 changes per second, we get 3×10^{26} states, which is a ridiculously small fraction of all possible states.

(c) Clearly the system does not access almost all of its "accessible" microstates. It may be better to interpret accessibility in terms of there being no distinction between actually accessed and accessible states, so all *can* be accessed, so all need to be taken into account.

28 With 52 cards, there are 52! possible arrangements. So

$$S = k \ln 52! = 156k = 2.16 \times 10^{-21} \text{ J/K}$$

This is a very small entropy. If $N \sim 10^{23}$ for the cards,

$$S \sim k(N \ln N - N) \sim 100 \text{ J/K}$$

There is no comparison.

35 The ideal gas entropy will become negative after

$$\ln\left[\frac{V}{N} e^{5/2} \left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right] + \frac{5}{2} = 0$$

Now use equipartition to write $U = \frac{3}{2}NkT$, and

$$1 = \frac{V}{N} e^{5/2} \left(\frac{2\pi m k T}{h^2}\right)^{3/2} \quad \Rightarrow \quad T = \left(\frac{N}{V}\right)^{2/3} \frac{h^2}{2\pi e^{5/3} m k}$$

Assuming ideal gas behavior at even very low temperatures, $N/V = P/kT$. Use $T = 300$ K and $P = 10^5$ Pa for ordinary conditions. Using the mass of Helium as 4u (atomic mass units), we get $T \approx 0.01$ K.