Solutions to Exam 1; Phys 186

1. (30 points) You need a diffraction grating that, when you shine white light through it, shows the full rainbow for visible colors three times on either side of the central white spot. How many lines/mm will your diffraction grating need to have to do the job? Is this number a minimum or maximum? Explain your reasoning.

Answer: Since $\sin\theta_m = m\lambda/d$, within the rainbow, the longer wavelengths will have the larger angle. Therefore, to just barely fit the third rainbow in, the red color, which is the longest visible wavelength, must be at an angle of $\theta_3 = 90°$. With $m = 3$ and $\lambda = 700\,\text{nm}$, we get

$$d = \frac{m\lambda}{\sin 90°} = \frac{3 \cdot 700}{1}\,\text{nm} = 2.1 \times 10^{-6}\,\text{m}$$

The line density is $1/d$, so this means a diffraction grating of $1/d = 476$ lines/mm.

Increasing the line density would make d smaller, which would increase the angle θ_3 beyond 90°, making at least part of the third rainbow invisible. Therefore, this line density is a maximum.

2. (40 points) In the physics lab, you notice that a spring you're playing around with has a period of exactly 2.00 seconds when you attach a mass hanger and three masses, each with a mass of 50.0 g, and extend the spring down by exactly 11.5 cm and let it go. You then think it would be amusing if you were to use the spring as an egg timer, and borrow the spring from me and take it home.

Next morning, you prepare to boil a three-minute egg. But you realize that you've forgot to borrow a ruler from the lab, and you have no way to measure the spring extension. Moreover, you brought the 50.0 g mass holder with you, but none of the masses. You're caught in a snow storm and there's no way to come down to campus and collect the equipment you need. Worse, you have nothing to keep time with where you live—your cell phone, for example, has a dead battery.

How, then, would you use the spring to time your three-minute egg? Describe exactly what you would do, using the appropriate equations when needed for your reasoning.

Answer: You check your physics notes and find that the period of a spring is $T = 2\pi/\omega = 2\pi\sqrt{m/k}$. This doesn't depend on the amplitude, so the extension of the spring when you release it is irrelevant. You don't need a ruler.

You then also notice that $T \propto \sqrt{m}$. If you use only the mass hanger, you will be oscillating the same spiring with 1/4 of the original mass attached. Therefore, your new period of oscillation will be $\sqrt{1/4} = 1/2$ times the original period.

You start the mass hanger bouncing around, confident that each period will now be $2.00/2 = 1.00\,\mathrm{s}$. Three minutes is 180 seconds, so you just need to count to 180 to get your egg just right.

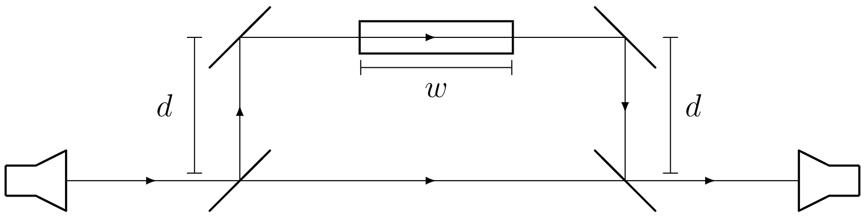
3. (40 points) When you try to figure out interference, you calculate the length difference between alternative paths from the source to the detector. But if you pass electromagnetic waves through a transparent object with index of refraction $n \neq 1$, the effective path length within the substance will change, because the number of wavelengths that fit into the object will be different compared to empty space (or air). For example, say the width of your object is $w = 12\lambda$, but the wavelength becomes smaller within the object so you fit 13 actual wavelengths into it. Then the effective width will be $w_{\text{eff}} = 13\lambda$.

(a) You have an object with index of refraction n and width w. What is its effective width for purposes of figuring out interference? Explain your reasoning. *Hint:* this isn't complicated.

Answer: Compared to space (or air), microwaves slow down in plastic, so that the speed in plastic $v_p = c/n_p$. Since the frequency does not change, and the speed of a wave is $v = \lambda f$, this means that microwaves in the plastic will have their wavelengths shortened to λ/n_p.

If the number of wavelengths that fit into a width w in empty space (or air) is w/λ, the number that will fit into a length w of plastic is $n_p w/\lambda$. Comparing the two expressions, we see that $w_{\text{eff}} = n_p w$.

(b) In Assignment 2, you had an interferometer, using microwaves with $\lambda = 3.00\,\mathrm{cm}$. You set up the same experiment, except that into the upper path, you insert a plastic object with width $w = 10.00\,\mathrm{cm}$. You then find that, with the object in place, the seventh point of constructive interference is no longer at $d = 10.50\,\text{cm}$, but has changed to $d = 10.00\,\text{cm}$. What is the index of refraction of your plastic object for 3 cm wavelengths?



Answer: We will need to use $l_1 - l_2 = m\lambda$ for constructive interference. But now, the path length difference will no longer be $2d$ with the plastic inserted. The effective length of the plastic will be $n_p w$, but that replaces a length in air of w. Therefore, the effect of inserting the plastic will be to add $(n_p - 1)w$ to the path length difference:

$$l_1 - l_2 = 2d + (n_p - 1)w = m\lambda$$

"The seventh point of constructive interference" means $m = 7$. Without the plastic, you'd have $2d = 7(3.00)$, to see constructive interference at $d = 10.5\,\text{cm}$. With the plastic, you see this at $d = 10$ cm. This means that

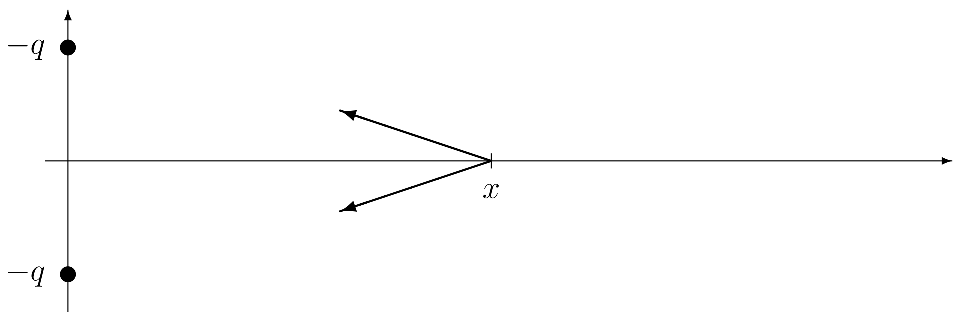
$$2d + (n_p - 1)w = m\lambda \quad \Rightarrow \quad 2(10\,\text{cm}) + (n_p - 1)(10\,\text{cm}) = 7(3\,\text{cm})$$

Solving this,

$$n_p = \frac{21 - 20}{10} + 1 = 1.1$$

So the index of refraction for microwaves of this plastic is 1.1.

4. (40 points) You have charges $-q$ at $x = 0, y = a$ and $x = 0, y = -a$. The positions of the charges are fixed.



(a) Find the total electric field at a point a distance x from the origin on the x axis. In other words, find the total E_x and E_y as functions of k, q, a, and x.

Answer: For the electric fields due to the individual charges we have, with the top charge 1 and the bottom number 2,

$$E_{1x} = -E_1 \cos\theta \qquad E_{1y} = E_1 \sin\theta$$

$$E_{2x} = -E_2 \cos\theta \qquad E_{2y} = -E_2 \sin\theta$$

where, as with examples you're familiar with, $E_1 = E_2 = kq/r^2$, $\cos\theta = x/r$, $\sin\theta = a/r$, and $r = \sqrt{x^2 + a^2}$. With these in place,

$$E_{1x} = -kq\frac{x}{(x^2 + a^2)^{3/2}} \qquad E_{1y} = kq\frac{a}{(x^2 + a^2)^{3/2}}$$

$$E_{2x} = -kq\frac{x}{(x^2 + a^2)^{3/2}} \qquad E_{1y} = -kq\frac{a}{(x^2 + a^2)^{3/2}}$$

Therefore, for the total electric field $\vec{E}$ we get

$$E_x = E_{1x} + E_{2x} = -2kq\frac{x}{(x^2 + a^2)^{3/2}}$$

$$E_y = 0$$

This is toward the left.

(b) When $x \gg a$ (x is much larger than a), $x + a \approx x$, $x^2 + a^2 \approx x^2$, and so on. Using such approximations, find out how the electric field behaves very far from the two charges. (*Hint:* This should be a mathematically simple expression.)

Answer: Using $x^2 + a^2 \approx x^2$,

$$E_x \approx -2kq\frac{x}{(x^2)^{3/2}} = -2kq\frac{x}{x^3} = k\frac{(-2q)}{x^2}$$

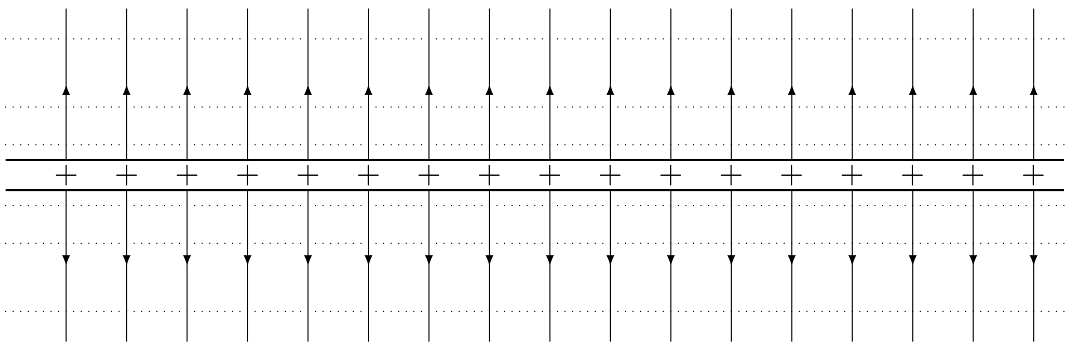
(c) Interpret your result for the electric field far away. Physically, *what does it mean?* Is it what you would expect?

Answer: Notice that your expression is the same as what you would get for a charge of $-2q$ at $x = 0$ and $y = 0$. And $-2q$ is the total charge we have on hand. Far away from a charge distribution, it behaves like a single charge equal to the total charge. That shouldn't be surprising.

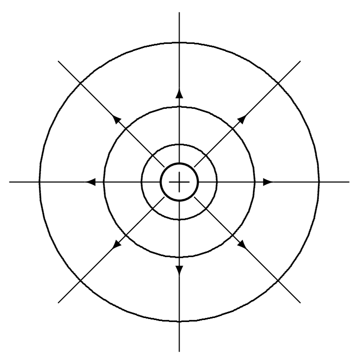
5. (50 points) You have a very long straight wire: an infinite wire, to keep things simple. You have charges on this wire, such that the amount of charge per unit length is a constant.

(a) Draw electric field lines for such a configuration; use solid lines. Then also draw equipotential (equal voltage) lines; use dashed lines or a different color. Note that electric field lines extend throughout three-dimensional space, and that in 3D, what we really have are equipotential *surfaces.* To indicate the 3D character of the lines, draw two pictures. The first is a side view of the wire. The second is a head-on view of the same wire: you're looking at the wire coming out of the page. In both cases, your electric field lines will be just those in the plane represented by the paper, and the equal voltage lines will indicate where the equipotential surfaces intersect the plane of the paper.

side view:



head-on view:



Answer:

(b) Recall how we established the distance dependence of an electric field due to a point charge and an infinite plane of charge by seeing how the field line density (field lines per area) changed with spheres or planes positioned at various distances from the point or plane charge. Let's do the same here. Pick an appropriate surface (tell me what it is) positioned at a fixed distance away from the line of charge. Call that distance r. Then, using how the line density charges, figure out the r-dependence of the electric field magnitude. Make appropriate drawings as part of your argument.

Answer: The same pictures before will do, except that instead of the equipotential surfaces, think of the lines as marking out concentric cylindrical areas centered on the wire. As the cylinders get larger in radius r, the number of electric field lines going through the cylinders will be constant. Therefore the line density, which is the number of lines going through an area, will be

$$E \propto \frac{\#\text{ lines}}{2\pi r h} \propto \frac{1}{r}$$

where h is the height of the cylinder, which is constant and therefore irrelevant.

Therefore, the electric field magnitude will fall off as the inverse distance to the wire.